

國立成功大學
111學年度碩士班招生考試試題

編 號： 110

系 所： 工程科學系

科 目： 訊號與系統

日 期： 0220

節 次： 第 2 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) For any discrete-time signal $x[n]$, show that

$$\sum_{m=-\infty}^n x[m] = x[n] * u[n]$$

where '*' denotes the convolution and $u[n]$ is the unit step function.

2. (30%) For two continuous-time signals $x(t)$ and $y(t)$, the cross-correlation function is defined as

$$\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t + \tau)y(\tau) d\tau.$$

Let $X(j\omega)$ and $Y(j\omega)$ be the spectra of $x(t)$ and $y(t)$, respectively. Also let $\Phi_{XY}(j\omega)$ be the spectrum of $\phi_{xy}(t)$ and $\Phi_{XX}(j\omega)$ be the spectrum of $\phi_{xx}(t)$.

- a. (10%) Show that

$$x(t) * y(-t) = \int_{-\infty}^{\infty} x(\tau)y(\tau - t) d\tau.$$

- b. (10%) Express $\Phi_{XY}(j\omega)$ in terms of $X(j\omega)$ and $Y(j\omega)$.

- c. (10%) If $x(t)$ is real, express $\Phi_{XX}(j\omega)$ in terms of $X(j\omega)$.

3. (30%) Consider a discrete-time system with input $x[n]$ and output $y[n]$. Let $y[n] = x_e[n]$ where $x_e[n]$ is the even component of $x[n]$.

- (5%) Determine the output of the system if the input is the impulse function $\delta[n]$.
- (5%) Is this system memoryless?
- (5%) Is this system causal?
- (5%) Is this system stable?
- (5%) Is this system linear?
- (5%) Is this system time-invariant?

4. (10%) Use Parseval's relation to find the value of

$$\int_{-\infty}^{\infty} \frac{\sin^2(\omega)}{\omega^2} d\omega.$$

5. (20%) Let $x(t) = e^{-a|t|}$ for $a > 0$.

- (10%) Find the Fourier transform $X(j\omega)$ of $x(t)$.
- (10%) Find the value of

$$\int_0^{\infty} \frac{2\pi}{\pi^2 + \omega^2} d\omega.$$