國立成功大學

113學年度碩士班招生考試試題

編　號：119

系　所：工程科學系

科　目：計算機數學

日　期：0202

節　次：第 3 節

備　註：不可使用計算機

編號： 119

國立成功大學113學年度碩士班招生考試試題

系　　所：工程科學系

考試科目：計算機數學

考試日期：0202，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Of 100 students in a university department, 45 are enrolled in English, 30 in History, 20 in Geography, 10 in at least two of three courses and just 1 student is enrolled in all three courses.
   (a) How many students take at least one of these courses? (3%)
   (b) How many students take none of these courses? (3%)
   (c) How many students take exactly one course? (3%)

2. A problem is given to three students whose chances of solving it are $1/2$, $1/3$ and $1/4$ respectively. What is the probability that the problem will be solved? (6%)

3. Two dice are tossed. What is the probability that the total score is a prime number? (6%)

4. Find the probability that the vowels in the word "AFFLIATION" will come together if the letters are randomly arranged in different ways. (8%)

5. Choose the words that correctly complete the following sentence: Suppose a 3 by 5 matrix A has rank $r = 3$. Then, the equation $Ax = b$ (always / sometimes but not always) has (a unique solution / many solutions / no solution). (6%)

6. Find the solution of second-order linear homogeneous recurrence relation
   $$a_n = 2a_{n-1} + 15a_{n-2}$$
   with $a_0 = 1$ and $a_1 = -4$. (12%)

7. Suppose A is the matrix
   $$A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 0 & 3 & 8 & 7 \\ 0 & 0 & 4 & 2 \end{bmatrix}.$$
   Find all special solutions to $Ax = 0$ and describe in words the whole nullspace of A. (10%)

國立成功大學 113 學年度碩士班招生考試試題

系　　所：工程科學系

考試科目：計算機數學

考試日期：0202，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

8. This question is about the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix}$$

(a) Find its eigenvalues and eigenvectors. (6%)

(b) Find the 3 matrices in the Singular Value Decomposition $A = U\Sigma V^T$ in two steps.

- First, compute V and Σ using the matrix A^TA.
- Second, find the (orthonormal) columns of U. (12%)

9. Suppose x_k is the fraction of NCKU students who prefer calculus to linear algebra at year k. The remaining fraction $y_k = 1 - x_k$ prefers linear algebra.

At year $k+1$, $1/5$ of those who prefer calculus change their mind. Also at year $k+1$, $1/10$ of those who prefer linear algebra change their mind.

Create the matrix A to give $\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = A \begin{bmatrix} x_k \\ y_k \end{bmatrix}$ and find the limit of $A^k \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as $k \to \infty$. (12%)

10. Graph G is represented by the following adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

(a) Draw the graph G. (4%)

(b) Determine whether G is a tree. Justify your answer. (3%)

(c) Determine whether G is an Eulerian graph. Justify your answer. (3%)

(d) Determine whether G is a Hamiltonian graph. If it is so, provide a Hamiltonian cycle on G. (3%)