國立成功大學

114學年度碩士班招生考試試題

編　號：96

系　所：工程科學系

科　目：計算機數學

日　期：0211

節　次：第 3 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. Approximate the root of $f(x) = x^3 - 6x + 1$ using Newton's method. Start with $x_0=2$ and perform two iterations. (15%)

2. Assume the passwords are selected from four-character combinations of 26 alphabetic characters. Assume that an adversary (i.e., a bad guy) can attempt passwords at a rate of one per second.
   (A) Assuming no feedback to the adversary until each attempt (of four characters) has been completed, what is the expected time to discover the correct password? (5%)
   (B) Assuming feedback to the adversary flagging an error as each incorrect character is entered, what is the expected time to discover the correct password? (5%)

3. Consider RSA with $p = 3$ and $q = 11$. Also, we have the following rules:
   (1) Compute $n = pq$, $z = (p - 1)(q - 1)$,
   (2) Choose e (with $e < n$) such that e and z are coprime,
   (3) Choose d such that (ed mod $z = 1$).

   (A) What are n and z? (6%)
   (B) Is $e = 5$ or $e = 7$ a better choice? Why? (3%)
   (C) Find d so its value is as small as possible. (3%)
   (D) Show the public and private keys. Given the message m=2, what is the corresponding ciphertext c? Show all work of encryption and decryption. (8%)

4. Given the dataset:

$$\mathbf{X} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 2.5 \end{bmatrix}.$$

   Use the normal equation $\mathbf{w} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$ to compute the linear regression coefficients. (10%)

5. Write an algorithm to determine if a point P(x, y) lies inside a triangle with vertices A(0, 0), B(4, 0), and C(0, 3). Test your algorithm with P(1, 1) and P(5, 5). (10%)

6. A box contains 3 red, 4 blue, and 5 green balls. One ball is drawn at random. If it is known that the ball is not red, what is the probability that it is blue? (5%)

7. A medical test for a disease gives a positive result 95% of the time if the person has the disease, and 10% of the time if the person does not. If 1% of the population has the disease, what is the probability that a person actually has the disease given that they tested positive? (10%)

8. A discrete random variable **X** has the following probability distribution:

$$P(\mathbf{X}=x) = \begin{cases} 0.2 & \text{if } x=1, \\ 0.5 & \text{if } x=2, \\ 0.3 & \text{if } x=3. \end{cases}$$

Find E(**X**) and Var(**X**). (10%)

9. Determine if the following graph has an Eulerian path or an Eulerian circuit:
Vertices: **V** = {A, B, C, D}
Edges: **E** = {AB, BC, CD, DA, AC}. (10%)