國立成功大學
114學年度碩士班招生考試試題

編　號：91

系　所：工程科學系

科　目：工程數學

日　期：0211

節　次：第 3 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

Notice: All questions **must be answered with detailed solution steps**. Otherwise, even if the chosen option is correct, no points will be awarded. 所有題目都要以詳細解題過程作答，否則即使選項正確也不算分數。

1. ( ) Solve the non-homogeneous ordinary differential equation (ODE)

$$y'' - 3x^{-1}y' + 4x^{-2}y = 2$$

Find the general solution $y(x) = ?$ **(20%)**

(A) $c_1x + c_2x^3 + 2x^2 + x - 2$, (B) $c_1x^{-1} + c_2x^3 + x^2\,ln\,x$

(C) $c_1x^3 + c_2x^{-1} + 3x^2 - 2x + 5$, (D) $c_1x^2 + c_2x^2 lnx + x^2\,(ln\,x)^2$

(E) $c_1x^{-4} + c_2x^2 + x^2\,ln\,x$, (F) $c_1x^{-1} + c_2x^{-2}lnx + x^{-2}\,ln\,x$

2. ( ) Use the Laplace transform to solve $y'(t) + 9\int_0^t y(\tau)\,d\tau = \sum_{k=1}^{\infty} cos\,k\,t\,\delta(t - k\pi)$ with the initial condition $y(0) = 0$, where $\delta(t)$ is the Dirac delta function. Determine which of the following is the solution of $y(t)$ on the interval $3\pi \le t < 4\pi$: **(20%)**

(A) $y(t) = 2\cos t \sin 2t$, (B) $y(t) = 3\cos t$, (C) $y(t) = 2\sin 2t$, (D) $y(t) = 0$,

(E) $y(t) = 3\cos 3t$, (F) None of the above.

3. ( ) Determine which of the following represents the N-term partial sum $S_N(x)$ of the Dirac delta function $\delta(x)$ expanded as a Fourier series on the interval $(-\pi, \pi)$: **(20%)**

(A) $S_N(x) = \frac{1}{2\pi}\frac{\cos[\frac{(N+1)x}{2}]}{\cos(\frac{Nx}{2})}$, (B) $S_N(x) = \frac{1}{2\pi}\frac{\sin[\frac{(2N+1)x}{2}]}{\sin(\frac{x}{2})}$,

(C) $S_N(x) = \frac{1}{2\pi} + \frac{1}{\pi}\frac{\cos[(N+1)x]}{\cos x}$, (D) $S_N(x) = \frac{1}{2\pi} + \frac{1}{\pi}\frac{\cos[\frac{(N+1)x}{2}]}{\cos(\frac{x}{2})}$,

(E) $S_N(x) = \frac{1}{2\pi} + \frac{1}{\pi}\frac{\sin[(N+1)x]}{\sin(Nx)}$, (F) None of the above.

4. ( ) Consider the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

in the domain $0 < x < L$. The initial conditions are:

$$u(x,0) = f(x), \frac{\partial u}{\partial t}(x,0) = g(x)$$

and the boundary conditions are $u(0,t) = u(L,t) = 0$. Solve the problem using the method of separation of variables and Fourier series. **(20%)**

(A) $u(x,t) = \sum_{n=1}^{\infty}[\frac{1}{L}\int_0^{\infty} f(x)\sin(\frac{n\pi x}{L})\,dx\cos(\frac{n\pi ct}{L}) + \frac{1}{n\pi c}\int_0^{\infty} g(x)\sin(\frac{n\pi x}{L})\,dx\sin(\frac{n\pi ct}{L})]\sin(\frac{n\pi x}{L})$,

(B) $u(x,t) = \sum_{n=1}^{\infty}[\frac{1}{L}\int_0^{L} f(x)\sin(\frac{n\pi x}{L})\,dx\cos(\frac{n\pi x}{L}) + \frac{1}{n\pi c}\int_0^{L} g(x)\sin(\frac{n\pi x}{L})\,dx\sin(\frac{n\pi x}{L})]\sin(\frac{n\pi ct}{L})$,

(C) $u(x,t) = \sum_{n=1}^{\infty}[\frac{2}{L}\int_0^{L} f(x)\sin(\frac{n\pi x}{L})\,dx\sin(\frac{n\pi ct}{L}) + \frac{2}{n\pi c}\int_0^{L} g(x)\sin(\frac{n\pi x}{L})\,dx\cos(\frac{n\pi ct}{L})]\sin(\frac{n\pi x}{L})$,

(D) $u(x,t) = \sum_{n=1}^{\infty}[\frac{2}{L}\int_0^{L} f(x)\sin(\frac{n\pi x}{L})\,dx\cos(\frac{n\pi ct}{L}) + \frac{2}{n\pi c}\int_0^{L} g(x)\sin(\frac{n\pi x}{L})\,dx\sin(\frac{n\pi ct}{L})]\sin(\frac{n\pi x}{L})$,

(E) $u(x,t) = \sum_{n=1}^{\infty}[\frac{1}{L}\int_0^{L} f(x)\sin(\frac{n\pi x}{L})\,dx\cos(\frac{n\pi ct}{L}) + \frac{1}{n\pi x}\int_0^{L} g(x)\sin(\frac{n\pi x}{L})\,dx\cos(\frac{n\pi ct}{L})]\sin(\frac{n\pi x}{L})$,

(F) None of the above.

5. ( ) Given the vector field $\boldsymbol{F} = (y^2z, xz^2, xy^2)$, compute the divergence of the field $\nabla \cdot \boldsymbol{F}$ **(10%)**

(A) 0, (B) x, (C) y^2, (D) xz^2, (E) xy^2, (F) None of the above.

6. ( ) Given the vector field $\boldsymbol{F} = (y^2z, xz^2, xy^2)$, compute the curl of the field $\nabla \times \boldsymbol{F}$ **(10%)**

(A) $\langle 2xz, -y^2, z^2\rangle$, (B) $\langle 2yz, 2xz, 2xy\rangle$, (C) $\langle 0,0,0\rangle$, (D) $\langle y^2, z^2, -y^2\rangle$, (E) $\langle yz, -xz, xy\rangle$,

(F) None of the above.