國立成功大學

115學年度碩士班招生考試試題

編　號：88

系　所：工程科學系

科　目：計算機數學

日　期：0204

節　次：第 3 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. Multiple Choice Questions (5% each, total 40%)

Select the single best answer for each of the following sub-questions.

1.1. Which of the following is the contrapositive of the statement: "If n^2 is an even integer, then n is an even integer"?

(A) If n is an even integer, then n^2 is an even integer.

(B) If n is an odd integer, then n^2 is an odd integer.

(C) If n^2 is an odd integer, then n is an odd integer.

(D) n is even if and only if n^2 is even.

1.2. Let $\boldsymbol{f}: A \to B$ and $\boldsymbol{g}: B \to C$ be two functions. If the composite function $\boldsymbol{g} \circ \boldsymbol{f}$ is an injection (one-to-one), which of the following statements must be true?

(A) $\boldsymbol{g}$ must be an injection.

(B) $\boldsymbol{f}$ must be an injection.

(C) Both $\boldsymbol{f}$ and $\boldsymbol{g}$ must be injections.

(D) $\boldsymbol{g}$ must be a surjection.

1.3. Using Fermat's Little Theorem, what is $2^{100} \pmod{101}$? (Note: 101 is prime).

(A) 1

(B) 2

(C) 100

(D) 0

1.4. A Bernoulli trial has a probability of success $p = 0.4$. If the trial is repeated independently 5 times, what is the variance of the total number of successes?

(A) 0.24

(B) 1.2

(C) 2.0

(D) 0.8

1.5. Suppose a 3x3 matrix $\boldsymbol{A}$ is diagonalizable and has eigenvalues $\lambda_1 = 1$, $\lambda_2 = 2$, and $\lambda_3 = 3$. If $\boldsymbol{B = A^2 - 3A + 2I}$, what is the determinant of $\boldsymbol{B}$?

(A) 0

(B) 2

(C) 6

(D) 12

1.6. What is the maximum number of edges in a simple undirected graph with 6 vertices and no cycles?

(A) 5

(B) 6

(C) 15

(D) 30

1.7. If an algorithm has a time complexity defined by the recurrence $\boldsymbol{T(n) = 2T(n/2) + n^2}$, what is the Big-O complexity of the algorithm?

(A) $O(n \log n)$

(B) $O(n^2)$

(C) $O(n^2 \log n)$

(D) $O(n^3)$

1.8. Boolean Algebra: Which of the following is logically equivalent to the Boolean expression $X \oplus (X \oplus Y)$?

(A) X

(B) Y

(C) $X \cdot Y$

(D) $X + Y$

Section 2: Structured Problem Solving (15% each, total 60%)

2. Consider a binary communication channel. Due to noise, a transmitted '0' is received as a '1' with probability 0.1, and a transmitted '1' is received as a '0' with probability 0.2. Suppose the transmitter sends '0' with probability 0.6 and '1' with probability 0.4.

   (A) Calculate the total probability that a '1' is received. (7%)

   (B) If a '1' is received, what is the probability that a '1' was transmitted? (8%)

3. A continuous random variable X has the probability density function (PDF) given by:

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 3 \\ 0 & otherwise \end{cases}$$

   (A) Determine the value of the constant k that makes this a valid PDF. (5%)

   (B) Find the cumulative distribution function (CDF), $F(x)$. (5%)

   (C) Calculate the expected value $E[X]$. (5%)

4. NCKU researchers are studying the movement of users between two social media platforms, A and B. Each month, 30% of users on A switch to B, while 20% of users on B switch to A.

   (A) Construct the transition matrix P for this system. (5%)

   (B) If the initial distribution is 100% of users on platform A, what will the distribution be after two months? (5%)

   (C) Find the steady-state distribution of users between the two platforms. (5%)

5. Solve the following non-homogeneous linear recurrence relation:

$$a_n - 3a_{n-1} + 2a_{n-2} = 3^n$$

   with initial conditions $a_0 = 1$ and $a_1 = 3$. (15%)