國立成功大學
115學年度碩士班招生考試試題

編　號：84

系　所：工程科學系

科　目：工程數學

日　期：0204

節　次：第 3 節

注　意：1.不可使用計算機
　　　　2.請於答案卷(卡)作答，於
　　　　　試題上作答，不予計分。

Notice: All questions **must be answered with detailed solution steps**. Otherwise, even if the chosen option is correct, no points will be awarded. 所有題目都要以詳細解題過程作答，否則即使選項正確也不算分數。

1. The ordinary differential equation (ODE) is

$$(6x^2 + 4axy)\,dx + (4bx^2 + 3y^2)\,dy = 0 \qquad a,b \in R$$

(a) What is the exact condition of the following ODE? **(5%)**

(A) $a = b/2$, (B) $b = a/2$, (C) $a = b/3$, (D) $b = a/3$, (E) $a = b/4$, (F) $b = a/4$

(b) Under the exact condition, please solve the general solution. **(10%)**

(A) $u(x,y) = x^3 + 3bx^2y + 2y^3 = c, \quad c \in R$

(B) $u(x,y) = x^3 + 4bx^2y + 3y^3 = c, \quad c \in R$

(C) $u(x,y) = 3x^3 + 4bx^2y + 2y^3 = c, \quad c \in R$

(D) $u(x,y) = 4x^3 + ax^2y + y^3 = c, \quad c \in R$

(E) $u(x,y) = 2x^3 + 2ax^2y + y^3 = c, \quad c \in R$

(F) $u(x,y) = x^3 + 3ax^2y + 4y^3 = c, \quad c \in R$

2. Solve the no-homogeneous Euler-Cauchy equation. **(20%)**

$$(x+1)^2y'' - 4(x+1)y' + 6y = x+1$$

(A) $y = c_1(x+1) + c_2(x+1)^2 + \frac{(x+1)}{3}$

(B) $y = c_1(x+1)^2 + c_2(x+1) + (x+1)$

(C) $y = c_1(x+1)^2 + c_2(x+1)^3 + \frac{(x+1)}{2}$

(D) $y = c_1(x+1)^3 + c_2(x+1) + \frac{(x+1)}{4}$

(E) $y = c_1(x+1)^3 + c_2(x+1)^2 + 2(x+1)$

(F) $y = c_1(x+1)^4 + c_2(x+1) + (x+1)$

3. Use the Laplace transform to solve the differential equation for a single loop LR-series circuit: $L\frac{di(t)}{dt} + Ri(t) = E(t)$ subjected to the initial current $i(0) = 0$ with the inductance L=1H, resistance R=1Ω, and impressed voltage $E(t)$, where $E(t)$ is the square-wave and has period $T = 2$. For $0 \le t < 2$, $E(t)$ can be defined by $E(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & 1 \le t < 2 \end{cases}$. Determine which of the following expressions corresponds to the solution of $i(t)$ on the interval of $0 \le t < 4$: **(20%)**

(A) $i(t) = \left(1 - e^{-(t-1)}\right)u(t-1) - \left(1 - e^{-(t-2)}\right)u(t-2) + \left(1 - e^{-(t-3)}\right)u(t-3)$,

(B) $i(t) = \left(1 - e^{-(t-1)}\right)u(t-1) + \left(1 - e^{-(t-2)}\right)u(t-2) - \left(1 - e^{-(t-3)}\right)u(t-3) + \left(1 - e^{-(t-4)}\right)u(t-4)$,

(C) $i(t) = \left(1 - e^{-(t-1)}\right)u(t-1) - \left(1 - e^{-(t-2)}\right)u(t-2) + \left(1 - e^{-(t-3)}\right)u(t-3) - \left(1 - e^{-(t-4)}\right)u(t-4)$,

(D) $i(t) = (1 - e^{-t}) - \left(1 - e^{-(t-1)}\right)u(t-1) + \left(1 - e^{-(t-2)}\right)u(t-2) - \left(1 - e^{-(t-3)}\right)u(t-3)$,

(E) $i(t) = -\left(1 - e^{-(t-1)}\right)u(t-1) + \left(1 - e^{-(t-2)}\right)u(t-2) - \left(1 - e^{-(t-3)}\right)u(t-3) + \left(1 - e^{-(t-4)}\right)u(t-4)$,

(F) None of the above.

4. Expand $f(x) = u(x) - u(x-\pi), 0 < x < 2\pi$ into a Fourier series, where $u(x)$ is the unit step (Heaviside) function. Which of the following represents the Fourier series of $f(x)$? **(15%)**

(A) $f(x) = \sum_{n=1}^{\infty} \frac{2(1-(-1)^n)}{n\pi} sin(nx)$,

(B) $f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{(1-(-1)^n)}{n\pi} sin(nx)$,

(C) $f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{(1-(-1)^n)}{n\pi} cos\left(\frac{nx}{2}\right)$,

(D) $f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(1 - \cos\left(\frac{n\pi}{2}\right)\right) sin\left(\frac{nx}{2}\right)$,

(E) $f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} sin\left(\frac{n\pi}{2}\right) cos\left(\frac{nx}{2}\right)$,

(F) None of the above.

5. Please find the Fourier coefficients of the function

$$r(t) = \begin{cases} -1, -\pi < t < 0 \\ 1, 0 < t < \pi \end{cases}$$

on the interval $[-\pi, \pi]$. **(10%)**

(A) $a_0 = 0, a_n = 0, b_n = \frac{2}{n\pi}, n = 1, 3, 5, \cdots$

(B) $a_0 = 0, a_n = 0, b_n = \frac{2}{n\pi}, n = 2, 4, 6, \cdots$

(C) $a_0 = 0, a_n = 0, b_n = \frac{4}{n\pi}, n = 1, 3, 5, \cdots$

(D) $a_0 = 0, a_n = 0, b_n = \frac{4}{n\pi}, n = 2, 4, 6, \cdots$

(E) $a_0 = \frac{2}{\pi}, a_n = \frac{4}{n\pi}, b_n = 0, n = 1, 3, 5, \cdots$

(F) $a_0 = \frac{2}{\pi}, a_n = \frac{4}{n\pi}, b_n = 0, n = 2, 4, 6, \cdots$

6. Based on the Fourier series of $r(t)$ solved in 1., find a general solution of the ODE $y'' + \omega^2 y = r(t)$ with $|\omega| \neq 1,3,5,\cdots$ **(10%)**

(A) $y = \sum_{n=1,3,5,\cdots} \frac{2}{n(\omega^2-n^2)\pi} \sin nt$

(B) $y = \sum_{n=2,4,6,\cdots} \frac{2}{n(\omega^2-n^2)\pi} \sin nt$

(C) $y = \sum_{n=1,3,5,\cdots} \frac{4}{n(\omega^2-n^2)\pi} \sin nt$

(D) $y = \sum_{n=2,4,6,\cdots} \frac{4}{n(\omega^2-n^2)\pi} \sin nt$

(E) $y = \sum_{n=1,3,5,\cdots} \frac{4}{n(\omega^2-n^2)\pi} \cos nt$

(F) $y = \sum_{n=2,4,6,\cdots} \frac{4}{n(\omega^2-n^2)\pi} \cos nt$

7. Evaluate the integral $\oint_C \frac{e^z}{z^n} dz$, where the contour C is the unit circle counterclockwise and n is any positive integer. **(10%)**

(A) $2\pi i$

(B) $\frac{2\pi i}{n!}$

(C) $\frac{2\pi i}{(n-1)!}$

(D) $2\pi i\, n!$

(E) 0

(F) This integral is undefined.