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1. GIVEN $f(x,y,z)=0$

SHOW THAT $\left.\frac{\partial x}{\partial y}\right)_z \left.\frac{\partial y}{\partial x}\right)_z = 1$, $\left.\frac{\partial x}{\partial y}\right)_z \left.\frac{\partial y}{\partial z}\right)_x \left.\frac{\partial z}{\partial x}\right)_y = -1$ 。

2. LEGENDRE ELLIPTIC INTEGRAL OF 2nd KIND $E(k,\phi)$ IS DEFINED AS $E(k,\phi) \equiv \int_0^\phi \sqrt{1-k^2\sin^2\phi}\, d\phi$, $0 \le k \le 1$

FIND THE PERIMETER OF AN ELLIPSE WITH SEMI-MAJOR AXIS a AND SEMI-MAJOR AXIS b, IN TERMS OF THIS ELLIPTIC INTEGRAL。

3. FROM THE FOURIER UNILATERAL INTEGRAL TRANSFORM PAIR

$$T[f(t)] \equiv \bar{f}(\omega) \equiv \int_0^\infty f(t)\, e^{-i\omega t} dt$$
$$T^{-1}[\bar{f}(\omega)] = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{f}(\omega)\, e^{i\omega t} d\omega$$

DEDUCE THE LAPLACE INTEGRAL TRANSFORM PAIR.

AND DERIVE THE FORMULA $\int_0^\infty \mathcal{L}[f(t)]\, ds = \int_0^\infty \frac{f(t)}{t} dt$ 。

4. SHOW THAT

$$\mathcal{L}\left[\int_0^t g(t-\lambda) f(\lambda) d\lambda\right] = \mathcal{L}\left[\int_0^t f(t-\lambda) g(\lambda) d\lambda\right] = \mathcal{L}[f(t)] \cdot \mathcal{L}[g(t)]$$

WHERE $\mathcal{L}$ IS LAPLACE TRANSFORM OPERATOR.

5. GIVEN A SINGLE PARAMETER FAMILY OF CURVES ON A GIVEN SURFACE BY $F(x,y,z)=0$, $G(x,y,z)=C$ 。

TRY TO SET UP THE DIFFERENTIAL EQUATIONS OF THEIR ORTHOGONAL TRAJECTORIES ON THE SAME SURFACE.

6. AN INSULATED ROD OF LENGTH ℓ, WITH TEMPERATURE u ATENDS AS $u(0,t)=0$, $u_x(\ell,t)=-h\,u(\ell,t)$

IF THE INITIAL TEMPERATURE DISTRIBUTION IS $u(x,0)\equiv f(x)$, $0<x<\ell$

FIND ITS TEMPERATURE $u(x,t)$ 。

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7. FOR A BILINEAR TRANSFORMATION $w \equiv f(z) \equiv \dfrac{az+b}{cz+d}$, $ad-bc \neq 0$

SHOW THAT $\dfrac{w_1-w_2}{w_1-w_3}\dfrac{w_3-w_4}{w_2-w_4} = \dfrac{z_1-z_2}{z_1-z_3}\dfrac{z_3-z_4}{z_2-z_4}$

FIND A BILINEAR TRANSFORMATION WHICH TRANSFORMS $z = 0, -1, \infty$, TO $w = -1, -2i, i$,

8. DERIVE THE NECESSARY CONDITIONS FOR 2nd ORDER MATRICES

$A \equiv \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$, $B \equiv \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$ WITH PROPERTY $AB = BA$.

9. GIVEN NON-COPLANAR VECTORS $\vec{a}, \vec{b}, \vec{c}$, FIND THEIR RECIPROCAL VECTOR SET.

10. FOR, $y'' + y' + xy = 0$, BY ASSUME SOLUTION LIKE $y = \sum_{n=0}^{\infty} A_n x^n$, GET THE RECURRENCE FORMULA FOR A_n 。 AND WRITE A_2, A_3, A_4 IN TERMS OF A_0 AND A_1