

① Use Green's theorem in plane of the form $\int_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy = \oint_C (M dx + N dy)$,
 derive the vector function form $\int_S (\nabla \times \vec{B}) \cdot \vec{n} dS = \oint_C \vec{B} \cdot d\vec{r}$

and the complex function form $\int_S \nabla B dS = \frac{1}{i} \oint_C B dz$

② The displacement $u(x,t)$ of a vibrating string with fixed ends satisfies
 $u_{tt} = a^2 u_{xx}$, $u(0,t) = u(l,t) = 0$

by setting $u(x,t) = X(x)T(t)$ and separating variables, express $u(x,t)$ as an infinite series if

$$u(x,0) \equiv f(x), \quad u_t(x,0) \equiv g(x)$$

③ given X_1, X_2, X_3 as linearly independent base vectors in $V_3(R)$ space, determine a set of orthogonal base vectors Y_1, Y_2, Y_3 in the same space.

④ Forward difference operator Δ and shifting operator E are related by $\Delta = E - 1$. Try to establish Newton's forward interpolation formula symbolically.

⑤ For function with single variable $f(x)$, the Taylor's series expansion is like $f(x+a) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} x^n$

try to express it into symbolic form, then extend it to

function with two variables like $f(x,y)$

⑥ solve $y^2 \left(\frac{dy}{dx} \right)^2 - 2xy \left(\frac{dy}{dx} \right) + 2y^2 - x^2 = 0$