

1. The block diagram of a control system is shown in Fig. 1.

- (i) For  $G(s) = \frac{K(s+a)(s+2)}{s(s^2-1)}$ , find the requirements on  $a$  and  $K$  so that the system is stable. Express the stable region in the  $K$ -versus- $a$  plane. (Use  $K$  as vertical and  $a$  as horizontal axis)

- (ii) For  $G(s) = \frac{K}{(s+2)^n}$ , how large can the constant  $K$  for each positive integer value of  $n$  if the overall system is to be stable?

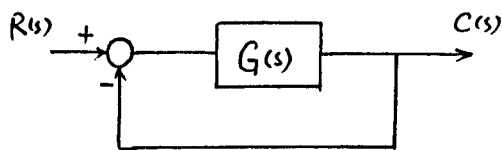


Fig. 1.

2. Find the transfer function  $C(z)/R(z)$  for the sampled-data system shown in Fig. 2, where  $G_1(s) = \frac{2}{s(s+1)}$ ,  $G_2(s) = \frac{1}{s+1}$ ,  $H(s) = \frac{1}{s}$ .

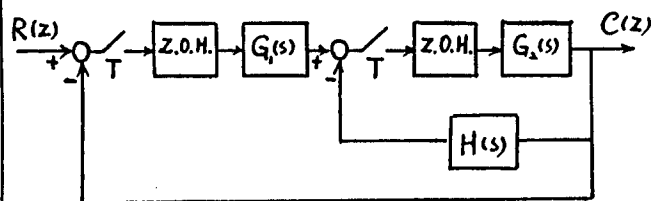


Fig. 2.

3. Given the  $n$ -th order system state equation:  $\dot{x}(t) = A x(t) + B u(t)$  output equation:  $C(t) = D x(t)$  where the pair  $[A, B]$  is completely controllable, and the pair  $[A, D]$  is completely observable. The closed-loop system obtained through

state feedback,

$$u(t) = r(t) - G x(t)$$

so that the state equation becomes

$$\dot{x}(t) = (A - BG) x(t) + B r(t)$$

- (i) Is the pair  $[A - BG, B]$  still completely controllable? prove it.  
 (ii) Is the pair  $[A - BG, D]$  still completely observable? prove it.

4. A system is shown in Fig. 3. The controlled process  $G_p(s)$  is modeled by the following equations:

$$\text{state equation: } \frac{dx_1(t)}{dt} = -2x_2(t)$$

$$\frac{dx_2(t)}{dt} = -2u(t)$$

$$\text{output equation: } c(t) = x_1(t)$$

- (i) The controller is the PD controller,

$$G_c(s) = K_p + K_D s$$

Find the values of  $K_p$  and  $K_D$  so that  $e_{ss} = 0$  when  $r(t) = \text{unit step function}$  and the poles of the closed-loop system are at  $-2$  and  $-2$ .

- (ii) Using the state-feedback control,  $u(t) = r(t) - G x(t)$ .

Find the feedback gain  $G$  so that minimize the performance index,  $J = \int_0^{\infty} [x_1^2(t) + x_2^2(t) + u^2(t)] dt$ .

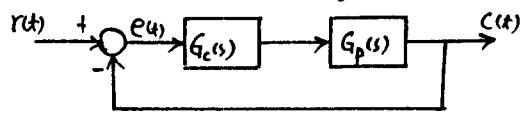


Fig. 3

5. Write dynamic equations for systems, each with modes  $e^{2t}$ ,  $e^{-3t}$ ,  $e^{(-4+j)t}$ , and  $e^{(-4-j)t}$  which have the following properties:

- (i) The mode  $e^{2t}$  is uncontrollable.  
 (ii) The mode  $e^{-3t}$  is unobservable.  
 (iii) The mode  $e^{2t}$  is both uncontrollable and unobservable.  
 (iv) The modes  $e^{(-4+j)t}$  and  $e^{(-4-j)t}$  are uncontrollable.  
 (v) The mode  $e^{-3t}$  is uncontrollable and the mode  $e^{2t}$  is unobservable.