1. The open-loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K(s+5)(s+40)}{s^3(s+200)(s+1000)}$$

(a) Discuss the stability of the closed-loop system as a function of K.
(b) Determine the values of K that will cause sustained oscillations in the closed-loop system. What are the frequencies of oscillations?

2. The block diagram of a digital control system is shown in Fig. 1. The controlled process is modeled by the following state equations:

$$\dot{x}_1 = -2x_2$$

$$\dot{x}_2 = -2u$$

and output equation is $c = x_1$. The controller is the digital PD controller with the transfer function:

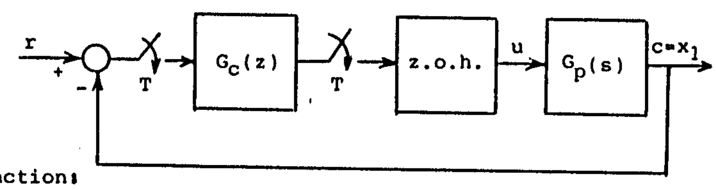

Fig. 1

$$G_c(z) = K_p + \frac{K_D(z-1)}{Tz}$$

Find K_p and K_D in terms of the sampling period T so that two of the roots of the closed-loop characteristic equation are at z = 0.5 and 0.5. Find the other characteristic equation root.

3. A control system is shown in Fig. 2. The system is assumed to be at rest initially. r(t) = a unit-step input. Determine the coefficient a so that $\int_0^\infty e^2(t)dt$ is minimized.

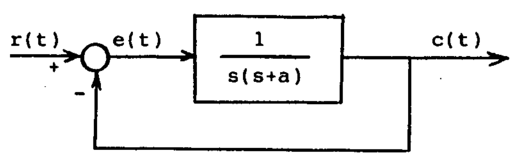

Fig. 2

4. A system is given by

$$\dot{X} = AX + Bu$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -500 & -150 & -20 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Apply state feedback u = -KX and use four different methods to find the elements of K so that the eigenvalues of the closed-loop system are at -5, -5+j5 and -5-j5.

5. A discrete-time system is described by the state equation: 113

$$X((k+1)T) = AX(kT) + Bu(kT)$$

where

X(kT) = nx1 state vector
u(kT) = control signal
A = nxn nonsingular matrix
B = nx1 matrix
T = sampling period

Prove that the system is completely state controllable if and only if the matrix $[B \;\; AB \;\; A^2B \;\ldots\ldots\; A^{n-1}B]$ is of rank n.