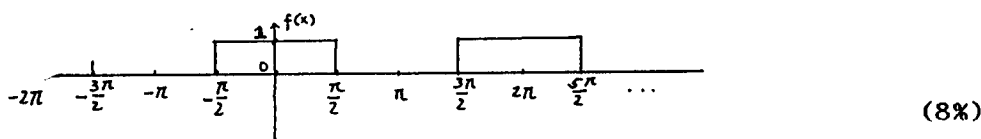1. Solve $y'' + 2y' + y = xe^{-x}$ (15%)

2. Find $\mathcal{L}^{-1}\left[\dfrac{s+1}{(s^2+1)(s^2+4s+13)}\right]$ (15%)

3. Let A and B be 3x3 matrices with det(A)=4 and det(B)=5. Find the values of
   (a) det(2AB) (3%)
   (b) det($A^{-1}B$) (2%)

4. Given $\dfrac{1}{C^2}\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$ with initial conditions
   $u(x,0) = f(x)$, $\dfrac{\partial u(x,0)}{\partial t} = g(x)$

   what's the solution of $u(x,t)$? (15%)

5. (a) Find the Fourier series of

   (8%)

   and (b) show that $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + - \cdots = \dfrac{\pi}{4}$ (7%)

6. Solve $\displaystyle\int_{-\infty}^{0}\dfrac{dx}{1+x^6} = ?$ (10%)

7. Differential equation $\dot{y} = Ay + h$

   and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ $A = \begin{bmatrix} 5 & 8 \\ -6 & -9 \end{bmatrix}$ $h = \begin{bmatrix} 1 \\ t \end{bmatrix}$

   (a) Find the eigenvalues of A. (5%)
   (b) Find the eigenvectors X of A. (5%)
   (c) Find $D = X^{-1}AX$. (5%)
   (d) Let $y = Xz$, and substituting into the differential equation, then $z = ?$ (5%)
   (e) If $y(0) = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$, then $y(t) = ?$ (5%)

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