1. (20%) Consider the following questions:
   (a) If the unit step response of a given system is $1 + 4e^{(-2t)} - 3e^{(-3t)}$, please find the transfer function of this system.
   (b) What is the output of the system found in part (a) if the input is $e^{(-2t)}$.
   (c) Find the transfer function, $G(s) = \frac{C(s)}{R(s)}$, of the system shown in Fig. 1.
   (d) Find the transfer function, $G(z) = \frac{C(z)}{R(z)}$, of the system shown in Fig. 2.

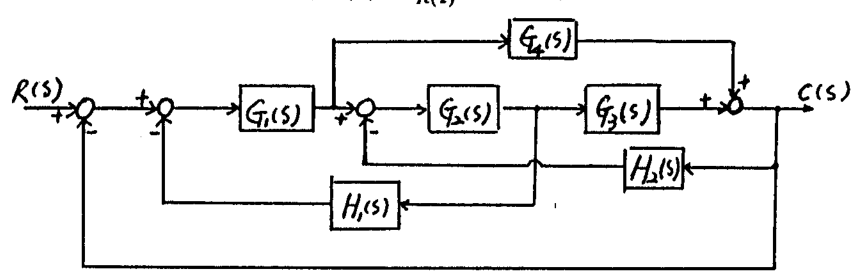

Fig. 1

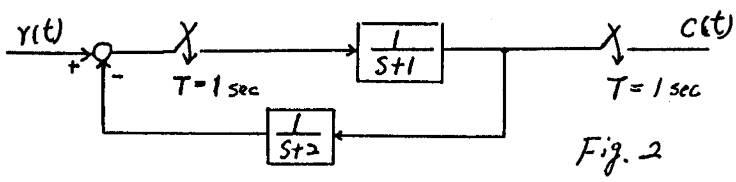

Fig. 2

2. (15%) A controlled system with a proportional controller is shown in the fig. 3.
   (a) Find the transfer function of the closed-loop system.
   (b) Find the value of the natural undamped frequency ω_n.
   (c) Find the value of the damping ratio ζ.
   (d) Find the value of K_p so that the steady-state error due to unit ramp input is 0.0001.

   If the above P controller is replaced with a PD controller, $K_p + K_d s$,
   (e) Find the value of K_p and K_d so that the ramp-error constant K_v is 1000 and the damping ratio is 0.5.

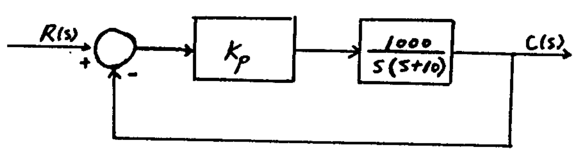

Fig. 3

3. (15%) The characteristic equation of a certain closed-loop control system is given by

$$s^3 + 3s^2 + (K+2)s + 3K = 0.$$

   (a) Determine the range of K so that the system is stable.
   (b) Determine the value of K that will cause sustained oscillation in the system. What is the frequency of oscillation ?

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4. A controlled system is described by the following state-space equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \qquad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$$

$$y = x_1.$$

(a) (5%) Find a state-feedback law $u = -k_1 x_1 - k_2 x_2 + r$ to move the poles to $s = -2 \pm j$.

(b) (5%) Find the region in the $k_1 - k_2$ plane on which the closed-loop system has $w_n \leq 10\ rad/sec$ and $\zeta \leq 0.6$.

(c) (10%) Find a state-feedback law $u = -k_1 x_1 - k_2 x_2$ to minimize the following performance criterion

$$J = \int_0^\infty (u^2(t) + y^2(t))dt.$$

5. (10%) Consider a body of unit mass moving along a line under the influence of a force u. Let $x_1(t) = y(t)$:its displacement at time t, and $x_2(t) = \dot{y}(t)$:its velocity at time t.

(a) Find the state-space equations of the system.

(b) Let $u = -y$, plot the state trajectory in the x_1-x_2 plane for $(y(0) = 1,\ \dot{y}(0) = 1)$.

(c) Apply the force $u = -sgn(y)$, and plot the state trajectory in the x_1-x_2 plane for the initial conditions given in part (b). Where

$$sgn(y) = \begin{cases} 1, & y > 0 \\ -1, & y < 0 \end{cases}.$$

6. (20%)

(a) If $\{A, b, c, d\}$, $d \neq 0$ is a realization with $H(s) = c(sI - A)^{-1}b + d$, show that $\{A - (bc/d), b/d, -c/d, 1/d\}$ is realization for a system with transfer function $\frac{1}{H(s)}$.

(b) Show that the zeros of $H(s)$ given in part (a) can be computed as the roots of the following equation

$$det \begin{bmatrix} sI - A & -b \\ c & d \end{bmatrix} = 0.$$