1. The linear system

E1: $30.00x_1 + 591,400x_2 = 591,700$

E2: $5.291x_1 - 6.130x_2 = 46.78$

Solve x_1 and x_2 with four-digit arithemetric and explain every-step which you write. (15%)

2. What's SOR (Successive-Over-Relaxation) method ?
Use the SOR method to solve the following linear system with two-digit arithemetric.

$$
\begin{aligned}
10x_1 - x_2 \quad\quad &= 9 \\
-x_1 + 10x_2 - 2x_3 &= 7 \\
-2x_2 + 10x_3 &= 6
\end{aligned}
$$

(15%)

3. We want to use the least squares approximations to $\sqrt{x}$ on $[0,1]$ by $a_0 + a_1 x = 0$, Find $a_0 = ?$, $a_1 = ?$. (15%)

4. We want to integrate $\int_a^b f(x)dx$ numerically by the two-term Gaussian quadrature i.e. $\int_a^b f(x)dx = c_1 f(t_1) + c_2 f(t_2)$. Find c_1, c_2, t_1 and t_2.

Use the above result to evaluate $\int_0^{\frac{\pi}{2}} \sin x dx$ and compare the exact result. (15%)

5. The initial-value problem is $\frac{dy}{dt} = f(t,y)$, $a \le t \le b$, $y(a) = \alpha$

(a) Write the difference equation by the Euler's method.

(b) Write its errors which are in the order of the step-size ' h' and explain the error bound when ' h ' decreases. (20%)

6. If we solve the linear equation $\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0$, u = constant > 0.

by the finite-difference method as $\frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} + u\frac{\phi_i^n - \phi_{i-1}^n}{\Delta x} = 0$.

Show that the method will render the difference equation back to the differential equation as $\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = \alpha\frac{\partial^2 \phi}{\partial x^2} + O(\Delta x^2, \Delta t^2)$.

Find $\alpha = ?$, Explain the physical meaning of the extra term. (20%)