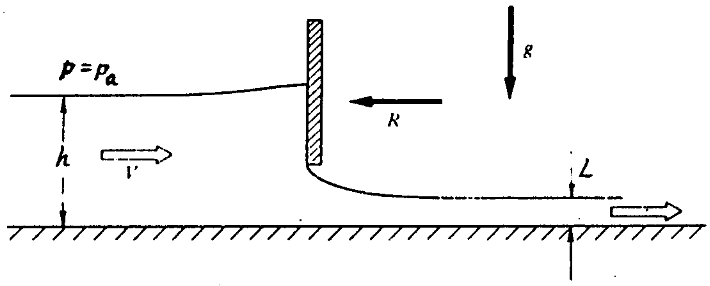
1 Under what conditions will the Continuity Equation reduce to $\nabla \cdot \vec{V} = 0$? What does this condition imply physically? 10%

2 Sketch neatly the streamlines for the two streamfunctions $\psi = X^2 + Y^2$ and $\psi = X^2 - Y^2$. Explain flow directions in both cases. 15%

3 What is a potential flow? Derive the governing equations for a two-dimensional, incompressible potential flow in Cartesian coordinates. 15%

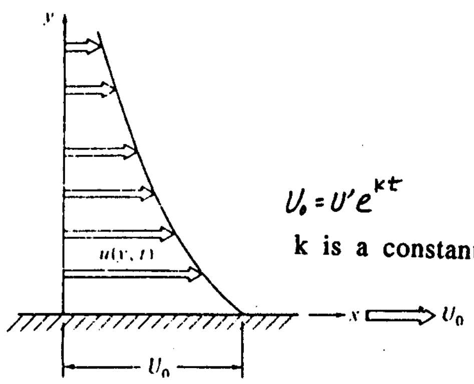
4 What are the key conditions for the existence of a (Prandtl) boundary layer? What are the important consequences of the boundary layer approximation? (You may use a flat plate problem to illustrate main features of boundary layer flows) 20%

5 An incompressible, nonviscous liquid of height 'h' and velocity 'V' flows under the action of gravity through a gate (shown below). The depth downstream of the gate is 'L'. Determine the force per unit width R necessary to hold the plate in place in terms of ρ (density), g, h, and L. 15%



6 A stationary plate of infinite length is immersed in a still (zero velocity) fluid of viscosity μ and constant density ρ. At time t = 0, the plate is made to move at the velocity $U_0(t)$, given below, in its own plane. The subsequent motion of the surrounding fluid u(y,t) is to be analyzed.

What are the initial and boundary conditions?
What are the governing equations (explain carefully why)?
Show how would you solve for the velocity distribution of this problem?
25%



$U_0 = U' e^{kt}$

k is a constant

useful equations:

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \frac{1}{3}\nu\frac{\partial \Theta}{\partial x} + f_x,$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \frac{1}{3}\nu\frac{\partial \Theta}{\partial y} + f_y,$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \frac{1}{3}\nu\frac{\partial \Theta}{\partial z} + f_z.$$

$$\Theta = \nabla \cdot \vec{V}$$