1. Suppose the matrix A ($=[a_{ij}]_{n\times n}$) is symmetric and whose eigenvalues λ_1, λ_2, ... and λ_n are distinct. Let V_1, V_2, ... and V_n be their corresponding eigenvectors.
   Given the vector equation Ax=b
   Questions:
   a. What is the form of b in terms of V_1,..and V_n? (7%)
   b. What is the solution of x in terms of V_1,.... and V_n ? (7%)

2. Show that if A is an nxn matrix with the property A{x} ={0} for all {x} $\in R^n$, then A is a zero matrix. (10%)

3. Given a matrix A and a vector b as

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -1 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix} \qquad b = \begin{Bmatrix} -2 \\ 0 \\ 8 \end{Bmatrix}$$

   Find the least squares solution of Ax=b. (10%)

4. Let $X=(x_1,....x_n)^T$ be an eigenvector of A belong to λ.

   If $|x_i|=\|X\|_\infty$, show that:

   (a) $\sum_{j=1}^{n} a_{ij}x_j = \lambda x_i$ (8%)

   (a) $|\lambda - a_{ii}| \le \sum_{j=1, j\neq i}^{n} |a_{ij}|$ (8%)

5. True or False (10%)
   (a) Is the graph whose adjacency matrix is given below plannar?

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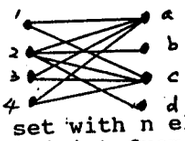
0 1 1 1 0 1
1 0 1 1 1 1
1 1 0 0 1 0
1 1 0 0 0 1
0 1 1 0 0 1
1 1 0 1 1 0
```

   (b) Can both Floyd's algorithm and Warshall's algorithm be used to test if a graph is connected?
   (c) Can a bipartite graph be a tree?
   (d) Is it true that the language defined by (x+y)* and the language defined by (x*y*)* are the same?
   (e) Is there a tree with 8 vertices of degree 1, 2 vertices of of degree 2, and 2 vertices of degree 3?
   (f) Let R be an equivalence relation on the set A, and let Ai be one of the equivalence classes. Can Ai be the empty set?
   (g) Consider a partial ordering and its Hasse diagram. If x is not directly connected to y in the Hasse diagram, then we do not have x < y.
   Let R and S be regular expressions. Answer the following:
   (h) Is RS = SR ?
   (i) Is R + S = S + R ?
   (j) IS R(S + T) = RS + RT?

6. Answer the following questions briefly.
(a) For the following bipartitie graph, either find a complete matching or show that none exists: (2%)



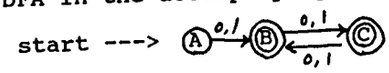
(b) Let A be a set with n elements and let B be a set with m elements, disjoint from A. If you have a graph G in which each edge joins an element of A to an element of B, what is the largest possible number of edges G can have? (2%)

(c) Simplify the Boolean expression
$f(x,y,z) = x\bar{z} + \bar{y}z + \bar{x}\bar{y} + \bar{x}yz$ (2%)

(d) A language L' defined by the following rules: a and ab are in L'; if xa is in L' then aaxbb is in L'; if axb is in L' then axa is in L'. How many strings of length 2k are there in L'? (2%)

(e) Find a regular expression for the language of all strings over {x,y} which begin with an x and end with a y. (2%)

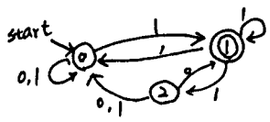
(f) Find a regular expression that represents the language accepted by the DFA in the accompanying figure: (2%)



(g) Construct a NFA from the following regular expression: (2%)
(0 + 1)* (000+11)

(h) If the state diagram for an NFA(Nondeterministic finite automaton) has n states, m of which are accepting, then how many states will the DFA(deterministic finite automaton) that simulates the NFA have? How many of them will be accepting (12%)

(i) Construct a DFA that simulates the NFA in the accompanying figure. (4%)



7. Find the solution to the HDE(Homogeneous Difference Equation):
$x_n - 4x_{n-1} + x_{n-2} + 6x_{n-3} = 0$
which satisfies the boundary conditions $x_0=6$, $x_1=9$, $x_3=87$ (10%)

8. Find the generating function for the sequence
0, 1, 0, 1, 0, 1, 0, 1, 0, 1, ... (5%)

9. A certain fast-food chain once offered the following promotional game: Tickets were issued, and each ticket contained seven "spots", coated with opaque paint. Two of the spots contained the words YOU WIN; two contained the word ZAP! (The other three spots contained things irrelevant to this discussion.) The customer was supposed to scrape the paint off of the spots, one at a time, the object being to uncover both YOU WIN spots before either ZAP! What is the probability of winning? (5%)