(1) We wish to solve f(x)=0 by using the $x_{n+1} = g(x_n)$ iteration.

(a) Analyze the iteration error and derive the sufficient condition for the method to converge.(10%)

(b) Use graphical representation to indicate two cases that the method would diverge and explain your reasons. (4%)

(2) We wish to solve f(x)=0 by using the Newton's iteration.

(a) Derive the expression for the iteration process.(5%)

(b) The iteration process of part (a) can find one root at a time. How would you obtain other roots if f(x)=0 has multiple roots? (5%)

(b) Use graphical representation to indicate three cases that the method would diverge and explain your reasons. (6%)

(3) Explain consistency, stability, convergence, and the Lax's equivalence theorem in the linear partial difference equation. (10%)

(4) Briefly describe the following numerical methods for solving a system of equation

Ax=b

where A is a n x n matrix, x and b are column vectors.

(a) The Gaussian elimination method (5%)

(b) The LU decomposition method (5%)

(c) The Gauss-Seidel iteration method (5%)

(d) The successive over relaxation method (5%)

5. We want to integrate $\int_a^b f(x)dx$ numerically by the two-term Gaussian quadrature i.e. $\int_a^b f(x)dx = c_1 f(t_1) + c_2 f(t_2)$. Find c_1, c_2, t_1 and t_2. Use the above result to evaluate $\int_0^{\frac{\pi}{2}} \cos x dx$ and compare the exact result. (20%)

6. If we solve the linear equation $\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0$, u = constant > 0. by the finite-difference method as $\frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} + u\frac{\phi_i^n - \phi_{i-1}^n}{\Delta x} = 0$. Show that the method will render the difference equation back to the differential equation as

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = \alpha\frac{\partial^2 \phi}{\partial x^2} + O(\Delta x^2, \Delta t^2).$$

Find $\alpha = ?$, Explain the physical meaning of the extra term. (20%)