

- (1) Define and explain the followings:(15%)  
 (a) Round-off error, (b) Truncation error, (c) Parallel processing, (d) Gauss-Seidel Iteration method, (e) The successive over relaxation method.
- (2) We wish to solve  $f(x)=0$  by using the Newton's iteration.  
 (a) Derive the expression for the iteration process.( 5%)  
 (b) The iteration process of part (a) can find one root at a time. How would you obtain other roots if  $f(x)=0$  has multiple roots? (4%)  
 (b) Use graphical representation to indicate three cases that the method would diverge and explain your reasons. ( 6%)
- (3) Use the LU decomposition method to solve the following equations. (20%)

$$\begin{cases} 2x_1 + 3x_2 - x_3 = 5 \\ 4x_1 + 4x_2 - 3x_3 = 3 \\ 2x_1 - 3x_2 + x_3 = 1 \end{cases}$$

- (4) We want to use the integral form of the least squares approximations to  $f(x) = x^{1.5}$  on  $[0,1]$  by  $p(x) = a_0 + a_1x + a_2x^2$ . Find  $a_0 = ?$ ,  $a_1 = ?$ ,  $a_2 = ?$  Compare  $f(x)$  and  $p(x)$  at  $x=0$ ,  $x=0.5$  and  $x=1$  respectively. (25%).

- (5) We want to solve  $\begin{cases} 2x_1 + x_2 = 24 \\ x_1 + 2x_2 = -12 \end{cases}$

Using the S.O.R. with a relaxation factor,  $\omega$ . Find the optimum value for this parameter and evaluate  $x^{(1)}$ ,  $x^{(2)}$ , and  $x^{(3)}$  using the zero vector as the start vector. (25%)

$$\left( \begin{array}{l} \text{Hint: } x^{(k+1)} = Bx^{(k)} + b' \\ B = (D + \omega L)^{-1}(D - \omega D - \omega U) \\ b' = (D + \omega L)^{-1} \omega b \end{array} \right)$$