1. Evaluate $\int_0^{\pi} \frac{d\theta}{a+b\cos\theta}$, $a > b > 0$. (10%)

2. Find the tangent plane and normal line to the surface $z = x^3y^5$ at the point (2, 1, 8). (10%)

3. Find the Laurent series of the function $z/(z^2 + z - 2)$ on the regions $0 < |z - 1| < 3$. (10%)

4. Given two matrices $M = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ and $K = \begin{bmatrix} 7 & 4 \\ 4 & 8 \end{bmatrix}$, there exists a matrix P such that

   $P^{-1}MP = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $P^{-1}KP = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

   Questions:

   (a) What are λ_1 and λ_2? (10%)

   (b) What is the matrix P? (10%)

5. Solve $\frac{dy}{dx} = \frac{1}{x + y^2}$ subject to $y(-2) = 0$. (10%)

6. Solve $x'' + 16x = \cos 4t$ subject to $x(0) = 0$, $x'(0) = 1$ by using Laplace transform. (12%)

7. Poisson's equation (16%)

   $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -h \qquad h > 0$

   Solve the equation subject to the conditions

   $u(0, y) = 0,\ u(\pi, y) = 1,\ y > 0$

   $u(x, 0) = 0,\ 0 < x < \pi$

   Hint: as $y \to \infty$, u has a finite value.

8. Expand $f(x) = x^2$, $0 < x < L$, (a) in a Fourier cosine series, (b) in a Fourier sine series. (12%)