⑧⑤學年度	國立成功大學 碩士班招生考試	工程科學 所	數值分析 試題	共 一 頁 第 一 頁

1. Lagrangian Polynomials can be used to interpolate a set of data points

   (x_i, f_i), $i = 0,1,2,......n$. Lets write the polynomial as $P_n(x) = \sum_{i=0}^{n} l_i(x) f_i$.

   Find $l_i(x)$=? (5%) Analyze the error caused by the interpolation. (5%).

2. We wish to solve the roots of f(x,y)=0 and g(x,y)=0,two nonlinear equations, using the Newton's iteration method,
   (a) Derive the expression for the iteration process.(10%)
   (b) use the result of part (a) to solve

   $$\begin{cases} x^2 + y^2 + xy = 28 \\ xy^2 = 16 \end{cases}$$ with the initial quess x=1 and y=1. (Do two iterations only).(5%)

3. We want to integrate the initial-value problem $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$ by the Runge-Kutta method (local error is 3th order, global error is 2rd order in step-size= h):(15%)

$$y_{n+1} = y_n + ak_1 + bk_2$$
$$k_1 = hf(x_n, y_n)$$
$$k_2 = hf(x_n + \alpha h, y_n + \beta k_1)$$

Prove that

$$a + b = 1,\quad \alpha b = 1/2,\quad \beta b = 1/2$$

4. Explain consistency, stability, convergence and the Lax's equivalence theorem for linear partial differential (difference) equation. Explain briefly how to prove consistency and stability problems for a finite difference scheme.(15%)

5 Draw a figure to explain what are dissipation and dispersion errors in numerical sense. (5%).

6. Give an example to explain what is explicit and implicit numerical schemes. Any advantage and disadvantage of the schemes? (5%)

7 Give examples to explain what is linear and nonlinear differential equations. How do you linearize a differential equation ? (5%)

8 Define and explain the followings:(10%)
(a) Round-off error, (b) Truncation error, (c) Gaussian elimination method, (d) Gauss-Seidel Iteration method, (e) The successive over relaxation method.

9. (a) The Cheybyshev polynomials $T_n(x)$ satisfies the following properties:

$$\int_{-1}^{1} w(x) T_n(x) T_m(x) dx = \begin{cases} 0 & n \neq m \\ \pi & n = m = 0 \\ 0.5\pi & n = m \neq 0 \end{cases}$$

where $w(x) = \frac{1}{\sqrt{1-x^2}}$, which is called weighting function. Prove that (10%)

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

(b) We use the Chebyshev polynomial expansion $y(x) = \sum_{i=0}^{n} a_i T_i(x)$ to approximate f(x) over [-1,1] with the weighting function w(x) as defined in part (a). Find a_i by using the integral form of the least-squares method. (10%)