1. Let V and V' be vector spaces with ordered bases $B=(b_1,b_2,b_3)$ and $B'=(b_1',b_2',b_3')$, respectively. Let $T:V\rightarrow V'$ be the linear transformation such that $T(b_1)=b_1'+2b_2'-3b_3'$, $T(b_2)=3b_1'+5b_2'+2b_3'$, $T(b_3)=-2b_1'-3b_2'-4b_3'$.

   Question:

   Express $T^{-1}(b_1')$, $T^{-1}(b_2')$ and $T^{-1}(b_3')$ as linear combinations of the vectors in B. (10%)

2. Find a basis for the subspace S of R^4 consisting of all vectors of the form $(a+b, a-2b+2c, b, c)$ where *a*, *b* and *c* are all real numbers. What is the basis for S and Dim(S)=? (5%)

3. Let *A* be an $n\times n$ matrix. The adjoint of *A* satisfies *(adj(A))A=A(adj(A))=det(A)I*, where *I* is the $n\times n$ identity matrix.

   Questions:

   (a) if *det(adj(A))*= $a(\det(A))^b$, then what are *a* and *b*? (5%)

   (b) if *adj(adj(A))*= $a(\det(A))^b A^c$, then what are *a*, *b* and *c*? (5%)

4. Let (a_1,b_1), (a_2,b_2),... (a_n,b_n) be data points. If the line that best fits the data in the least-squares sense is given by $cx+d$, then what are *c* and *d*? (10%)

5. Given a matrix $A=\begin{bmatrix}1&4&7\\2&5&8\\3&6&9\end{bmatrix}$

   Questions: find.

   (a) a basis of the column space of *A* and determine the rank of *A* (7%)

   (c) a basis of the null space of *A* and determine the nullity of *A* (8%)

6. Let A = {1,2,3,4}, please answer the following questions:
   (1) How many of those relations on A are both symmetric and antisymmetric?(3%)
   (2) How many different partial orderings are there on A?(3%)
   (3) How many different equivalent relations are there on A?(3%)

7. (1) Using the (7,4) Hamming code, encode the data word 0011.(3%)
   (2) Find a formula for the number of directed graphs (with no loops or multiple edges) having n vertices.(3%)
   (3) Let G be a connected graph with edge weights assigned. Find sufficient (but not necessary) conditions on G which will guarantee that G has a unique minimal spanning tree. (3%)
   (4) Given a connected undirected graph G with edge weights assigned. Suppose that a Dijkstra spanning tree,T, is constructed for some source node A. Is T a minimal spanning tree? Please prove or disprove it. (3%)
   (5) Simplify the Boolean expression F(w,x,y,z) =wxy'z + xy'z' + wx'z' + x'y'z'(3%)
   (6)Here is a program listing for a Turing Machine. If this machine is started in state S reading the leftmost x on a tape that is blank except for a string of x's, what will it do? (3%)

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Sx: AxR
Ax: AyR
Ab: AbH
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8. Refer to languages over the alphabet {x,y}.

(1) Give a regular expression for the language which accepts those strings either start with a x or contain a yy. (3%)

(2) Construct a nondeterministic finite automata which accepts the language in (1). (3%)

(3) Construct a deterministic finite automata which accepts the language in (1)(4%).

9. Find the general solution to the HDE (Homogeneous Difference Equations):

$$X_n - 6X_{n-1} + 12X_{n-2} - 8X_{n-3} = 0$$

with the boundary conditions $X_0 = 1$, $X_1 = 0$, $X_3 = -160$.(note: It is X_3, not X_2)(5%)

10. Let Bn be a sequence of non-negative integers satisfying the third order IDE (Inhomogeneous Difference Equations) : $B_0 = B_1 = 0$; $B_2 = 2$;

$$B_n - 2B_{n-1} - B_{n-2} + 2B_{n-3} = \begin{cases} 1 & \text{for } n = 3 \\ \text{or } 0 & \text{for } n \geq 4 \end{cases}$$

Find the generating function for the sequence B_n (5%) and derive an exact formula for the nth term in the generating function (3%).