1. $y = e^x$ is one solution of the homogeneous equation

$$y'' - 2y' + y = 0$$

Using $y = e^x$ to find a complete solution of the nonhomogeneous equation

$$y'' - 2y' + y = xe^x \qquad (10\%)$$

2. (a) Find the complete solution of

$$\frac{d^2}{dt^2}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f(t) \\ 0 \end{Bmatrix},$$

$$\frac{d}{dt}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}(0) = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix},\ \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}(0) = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \qquad (12\%)$$

(b) Let $f(t) = F(t)$, $0 \le t \le 1$. What is the expressions of $x_1(t)$ for $t > 1$. (8%)

3. (a) Given the differential equation

$$\frac{d^2 y}{dx^2} + \lambda p(x) y = 0,\quad 0 \le x \le 1,\ y(0) = y(1) = 0$$

where $p(x)$ is continuous and positive. Let $\lambda_1, \lambda_2, \ldots$ be distinct eigenvalues of the parameter λ for which this equation has corresponding eigenfunctions $y_1, y_2, \ldots$.

(1) show that any two distinct eigenfunctions are orthogonal over the interval $[0,1]$. (5%)

(2) using the result of (1) to find λ_i. (5%)

(b) Using the method of eigenfunction expansion to solve the problem

$$\frac{d^2 y}{dx^2} + \lambda p(x) y = f(x),\quad 0 \le x \le 1,\ y(0) = y(1) = 0$$

where $f(x)$ is continuous over the interval $[0,1]$. (10%)

(背面仍有題目,請繼續作答)

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10%

4. Evaluate $\int_c (6xy - 4e^x)dx + 3x^2 dy$

c: two lines connecting points (0,0), (-2,0) and (-2,0), (-2,1).

15%

5. $\oint_c \frac{\sin z}{Z^2(Z^2+4)} dz$

(a) c: is any closed path containing 0, 2i, -2i.

(b) c: $|z| = 1$.

25%

6. Consider a slender long rod composed of a uniform heat conducting material. We assume that heat can enter and leave the rod only through its ends. Find the time-dependent temperature distribution along the rod for $t > 0$, given (you should at least show major steps how to solve it)

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$T(0,t) = T_0$, $T(L,t) = T_1$, $\alpha > 0$, T_0 and T_1 are constants.

$T(x,0) = f(x), 0 \le x \le L$.