1. A binary system (二進位) is usually used in a computer, please convert the following binary expression $(1001.01)_2$ into the decimal (十進位) expression. What is its floating-point value? What is its fixed-point values? If we want to express the true value of 1/3 in a finite bit (say 32 bit) of a computer, what error will be caused? Why? (20%)

2. A quadratic spline is a continuous curve that has a continuous first derivative for a pair of second degree polynomials joining at each point. We wish to use a periodic quadratic spline to interpolate a set of periodic data given in one period as (x_0, f_0), (x_1, f_1), ..., (x_n, f_n) where $x_0 < x_1 < x_2 < .... < x_n$ and $f_0 = f_n$ the interval spacing is constant , i.e. $x_i - x_{i-1} = h$. Represent the quadratic in $[x_i, x_{i+1}]$ as $P(x) = a_i(x - x_i)^2 + b_i(x - x_i) + c_i$. Derive the periodic quadratic spline relations and indicate how to solve for a_i, b_i, and c_i ? (20%)

3. Briefly describe the following numerical methods for solving a system of equation Ax=b
   (a) The Gaussian elimination method (5%)
   (b) The LU decomposition method (5%)
   (c) The Gauss-Seidel iteration method (5%)
   (d) The successive over relaxation method (5%)

4. Consider the following differential equation:
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$
   (a) Use any explicit finite-difference method to discretize the equation.(5%)
   (b) Use any implicit finite-difference method to discretize the equation.(5%)
   (c) What's the major differences between the two methods? (3%) How about the numerical stability problem? (2%)

5. Consider a 2-D PDE $\frac{\partial T}{\partial t} = \alpha(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2})$. We would like to use the approximate-factorization method (or the so-called ADI method in delta form) to solve the above equation. (a) Derive the discretized equation that can be written as (use first order Euler backward in time):

$$(I - \Delta t\alpha\frac{\partial^2}{\partial x^2})\Delta T_{ij}^{*} = \Delta t\alpha(\frac{\partial^2}{\partial x^2}T_{ij}^{n} + \frac{\partial^2}{\partial y^2}T_{ij}^{n})$$
$$(I - \Delta t\alpha\frac{\partial^2}{\partial y^2})\Delta T_{ij}^{n+1} = \Delta T_{ij}^{*}$$
, where $\Delta T_{ij}^{n+1} = T_{ij}^{n+1} - T_{ij}^{n}$. (5%)

(b)What is the major advantage in using the above form ? (3%)
(c)What error will be produced by using and without using the above form? (2%)

(背面仍有題目,請繼續作答)

(d)Fill out the following matrix coefficients for the points a, b, c, d, e, f for the time level from n to n+1 (Use central difference for space and Euler backward for time, consider $\Delta x = \Delta y = 1$). Boundary conditions are shown in the figure. (You do not have to solve the system of equations).(15%)

$$\begin{bmatrix} a_{21} & a_{31} & & & & \\ a_{12} & a_{22} & a_{32} & & & \\ & a_{23} & a_{33} & a_{43} & & \\ & & a_{34} & a_{44} & a_{54} & \\ & & & a_{45} & a_{55} & a_{65} \\ & & & & a_{56} & a_{66} \end{bmatrix} \begin{bmatrix} \Delta T_a^* \\ \Delta T_b^* \\ \Delta T_c^* \\ \Delta T_d^* \\ \Delta T_e^* \\ \Delta T_f^* \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{bmatrix},$$

$$\begin{bmatrix} b_{21} & b_{31} & & & & \\ b_{12} & b_{22} & b_{32} & & & \\ & b_{23} & b_{33} & b_{43} & & \\ & & b_{34} & b_{44} & b_{54} & \\ & & & b_{45} & b_{55} & b_{65} \\ & & & & b_{56} & b_{66} \end{bmatrix} \begin{bmatrix} \Delta T_a^{n+1} \\ \Delta T_d^{n+1} \\ \Delta T_b^{n+1} \\ \Delta T_e^{n+1} \\ \Delta T_c^{n+1} \\ \Delta T_f^{n+1} \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix}$$