1. Let $y_1=(1,1,1)^T$, $y_2=(1,1,0)^T$, $y_3=(1,0,0)^T$, $e_1=(1,0,0)^T$, $e_2=(0,1,0)^T$, $e_3=(0,0,1)^T$ and I be the identity operator on R^3

   (a) Find the coordinates of $I(e_1)$, $I(e_2)$ and $I(e_3)$ with respect to $[y_1,y_2,y_3]$ (8%).

   (b) Find a matrix A such that Ax is the coordinate vector of x with respect to $[y_1,y_2,y_3]$ (10%).

2. Let S be the subspace of R^4 spanned by $x_1=(0,1,0,1)^T$ and $x_2=(1,0,-1,0)^T$

   (a) Find a basis $\{x_3,x_4\}$ for S such that $\{x_1,x_2,x_3,x_4\}$ is an orthogonal basis of R^4 (8%).

   (b) Express $y=(1,2,3,4)^T$ into the combination of x_1, x_2, x_3 and x_4 (10%).

3. Let x and y be distinct vectors in R^n with $\|x\|_2=\|y\|_2$. Define

$$u=\frac{1}{\|x-y\|_2}(x-y) \text{ and } Q=I-2uu^T.$$

   Show that

   (a) $\|x-y\|_2^2=2(x-y)^T x$ (7%).

   (b) $Qx=y$ (7%).

4. Prove the following statements if they are true. Disprove or show an counter example, if it is false.(20%)

(1) The language defined by (x + y)* and the language defined by (x*y*)* are the same.

(2) A relation can not be both symmetric and antisymmetric.

(3) Let F be the set of all real-valued functions on the set A. If f, g belongs to F, write $f \le g$ if $f(x) \le g(x)$ for all x belongs to A. The relation '$\le$' is a total ordering.

(4) If a graph has one more vertex than edges, then it is a tree.

(5) Suppose that G=(V,E) is a loop-free planar graph. Suppose further that G is planar and determines 53 regions. If, for some planar embedding of G, each region has at least five edges in its boundary, then the number of vertices must be grater or equal than 82.

5. Answer the following questions briefly.

Multiple choice:

(1) If m pigeons are placed in n pigeonholes, and m < n,

(a)Some pigeonhole must contain more than one pigeon.

(b)Some pigeonhole must be empty.

(c) Two pigeons must contain the same number of pigeons.

(d) Every pigeonhole must be occupied. (3%)

(背面仍有題目,請繼續作答)

(2) Which of the following identities for regular expressions is (are) true?

(a) $(r + s)^*r = r(sr + r)^*$

(b) $(r + s)^* = r^* + s^*$

(c) $s(rs + s)^* r = rr^*s(rr^*s)^*$ (3%)

(3) If the state diagram for an NFA(nondeterministic finite automata) has n states, m of which are accepting, how many states will the DFA(deterministic finite automata) that simulates the NFA have? How many of them will be accepting state?(4%)

(4) A certain language contains the five strings, a, bcd, cde, dcd, edc, and all strings that can be built by concatenating these words together. If Xn is the number of strings in the language of length n, set up a difference equation for Xn(5%).

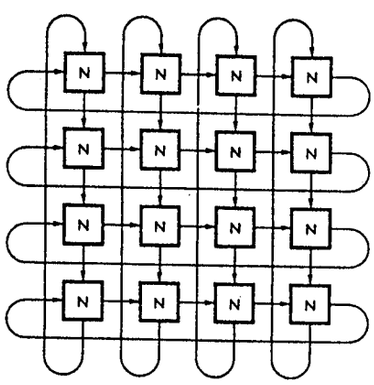
(5)In a network of computers, there are 25 computers, including 4 servers. In how many ways can a group of 4 members be chosen if exactly one server must be in a group.(5%)

6. A block diagram of a 2-dimentional message passing concurrent computer is shown in the following figure. The topology of the interconnection network is called a 2-dimensional torus. Each node consists of a processor, its local memory and a message-passing interface, generally called a router.

(1) Is this graph planar? Prove or disprove it.(3%)

(2) Is there an Euler cycle or path in this graph? If there is such a path or cycle please draw it.(2%)

(3) The 2-dimentional torus can be constructed by connecting the edge nodes of a 2-dimensional mesh. Connections on the edges of the torus wrap around to the opposite side. You would notice that there are some long end-around connections. Please find an isomorphism of the figure, which can avoid the long end-around connections. That is, try to make all the connections (edges) of the same length, and make the edges as short as possible. (5%)



A Torus System