1. $\frac{d^2y}{dt^2}+9y=f(t)$, $f(t)=\begin{cases}\sin 2t, & 0\le t\le \pi/2\\ 0, & \pi/2\le t\end{cases}$, $y(0)=\frac{dy(0)}{dt}=1$.

Find $y(3\pi/2)$ (15%).

2. $\frac{d^2y}{dx^2}+(\lambda\pi)^2y=f(x)$, $0\le x\le 1$, $y(0)=a$, $y(1)=b$.

Find $y(x)$ (15%).

3. Given the eigenvalue problem $\lambda Ax=Bx$, $A=\begin{bmatrix}4 & 0\\ 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}8 & 2\\ 2 & 8\end{bmatrix}$.

Find
(a) eigenvalues λ_1 and λ_2 (10%).
(b) a matrix P and such that $P^{-1}AP=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $P^{-1}BP=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}$ (10%).

4. Use Stokes's theorem to evaluate $\oint_C F\cdot dx$, where $F(x,y,z)=(3-2y)i+(3x-4y)j+(z+3y)k$ and C is the unit circle in the plane $z=2$. 15%

5. Calculate the eigenvalue and eigenfunction for 20%

$$\frac{\partial^2 u}{\partial t^2}=c^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)$$

B.C. $u(R,t)=0$

Explain how to determine the solution when I.C. is given

I.C. $u(r,0)=f(r)$

$u_t(r,0)=g(r)$

6. Evaluate $\int_0^{2\pi}\frac{\cos\theta}{3+\sin\theta}\,d\theta$ 15%