1. 20%

We want to integrate $\int_a^b f(x)dx$ numerically by the two-term Gaussian quadrature i.e. $\int_a^b f(x)dx \approx c_1 f(t_1) + c_2 f(t_2)$. Find c_1, c_2, t_1 and t_2. (10%) Use the above result to evaluate $\int_0^1 \int_0^{x^2+1} xy e^y dy dx$ and compare the exact result. (10%)

2. 20%

Suppose for Ax=b, that $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 3 & 4 \\ 2 & 4 & a \end{bmatrix}$

What is the smallest value of a for which convergence will be obtained by
(a) using Jacobi method? (10%)
(b)using Gauss-Seidel method? (10%)

3. 15%

The power method can be used to find the largest eigenvalue and its corresponding eigenvector in a system of algebraic equations. (a) Explain the theoretical background of the power method.(10%) (b)Find the dominant eigenvalue and the corresponding eigenvector by the power method for the matrix: $\begin{bmatrix} 3 & 1 \\ 2 & 9 \end{bmatrix}$ (5%).

4. 10%

The least-square method can be used to find an polynomial function to approximate a set of discrete data. Please explain how to find this function ? (5%). The least-square method can be generalized to an integral form of the least-square method, How? Give an example to explain it (5%).

5. 20%

(a) Lagrangian Polynomials can be used to interpolate a set of data points

(x_i, f_i), $i = 0,1,2,\ldots\ldots n$. Lets write the polynomial as $P_n(x) = \sum_{i=0}^{n} l_i(x) f_i$.

Find $l_i(x)$=? (5%) Analyze the error caused by the interpolation. (5%).

(b) We can use divided difference to obtain interpolation polynomial as:

$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \ldots\ldots + a_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$,

Find a_i= ? i=1,2....n. (5%)

(c) We can use either Lagrangian polynomial or divided difference in polynomial interpolation. Would the two resulting polynomials in (a) and (b) be different? Why? Explain your answer.(5%).

6. 15%

We can use cubic spline to do interpolation for a set of data given as
(x_0, f_0), (x_1, f_1), ..., (x_n, f_n) where $x_0 < x_1 < x_2 < \ldots < x_n$
The interval spacing is constant , i.e. $x_i - x_{i-1} = h$. Please indicate the major properties of the spline which joing the neighboring points. (6%). Can the spline relation be applied at end points (5%)? How do you handle these end points ? (4%). (Please no need to derive the spline relations.)