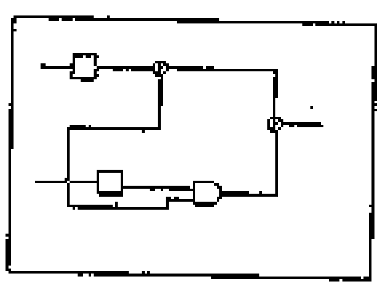
1. In R^4 let U be the subspace of all vectors of the form $(u_1, u_2, 0, 0)^T$ and let V be the subspace of all vectors of the form $(0, v_2, v_3, 0)^T$.
What are the dimensions of U, V, $U \cap V$, $U + V$? Find a basis for each of these four subspaces. (20%)

2. Let L be a linear operator mapping a vector space V into itself.
Define L^n, $n \geq 1$, recursively by
$L^1 = L$, $L^{k+1}(v) = L(L^k(v))$ for all $v \in V$ (15%).
show that L^n is a linear operator on V for each $n \geq 1$.

3. Find the eigenvalues λ_i and the corresponding eigenvectors v_i of the linear transformation T defined on R^3 by
$T([x_1, x_2, x_3]) = [3x_1 - x_2 + x_3,\ -2x_1 + 2x_2 - x_3,\ 2x_1 + x_2 + 4x_3]$ (15%).

4. (1) A transitive, and symmetric relation is reflexive. Prove or disprove it.(5%)
(2) No cycles can appear in any poset (partially ordered set). (Hint: You can use mathematical induction. Suppose the relation R is over some finite set { x_1, x_2, x_3,...,x_n} You should prove that if x R x_1 R x_2 R ...R x_n R x, then x = x_1 = x_2 = ... = x_n = x.)(5%)

5.(1) If in the following sequential circuit, both D flip-flops are initialized to 0, what is the sequence of outputs corresponding to the inputs (1,1), (1,0), (0,0), (0,1)? (Here the first coordinate corresponds to the top wire.)(3%)



(2) Simplify the Boolean expression f(x,y,z) = yz' + xz + y'z + x'yz(3%)
(3) Find a deterministic finite automata over {a,b} which simulates the nondeterministic finite automata. Here the start state is 1, and 1 and 3 are accepting states.(4%)

Present State / Next State

	X=a	X=b
1	1,2	2
2	3	2
3	---	---

(背面仍有題目,請繼續作答)

6.(1) A language contains the words ab, cd, efg, and xyz. Find a difference equation (including boundary conditions) for the number of sentences of length n. (5%)

(2)Suppose that the alphabet is {x,y}. Construct a nondeterministic finite automata which accepts the language consisting of all strings which contain the substring xxy.(5%)

(3) The game of Nim is a game played between two players, using piles of counters. In Nim, the players alternate the turns. When your turn comes, you must remove some (or all) of the counters from just one of the piles. The object is to be the one who removes the last counter from the table. Suppose we have 4 piles of counters, and we denote it as [11,13,6,20]. (That is the first pile has 11 counters, the second 13, the third 6, and the 4th is 20.) Find a good move. (5%)

7.(1) Is the graph whose adjacency matrix is given below planar?(2%)

0	1	1	1	0	1
1	0	1	1	1	1
1	1	0	0	1	0
1	1	0	0	0	1
0	1	1	0	0	1
1	1	0	1	1	0

(2)Show that every tree with at least two vertices has at least one vertex of odd degree.(4%)

(3)Show that a connected graph with n vertices and at least one cycle must have at least n edges.(4%)

(4) The complete bipartite graph $K_{m,n}$ is defined to be a bipartite graph with $|A| = m$, $|B| = n$, with every A-vertex connected to every B vertex.

(a) How many edges are in a maximal matching?(2%)

(b) When is there a complete matching?(3%)