1. Try an initial value $x^{(0)}$ to solve the equations of the system

$$x^{(i+1)} = Ax^{(i)} + b,\ \ x^{(i)}, b \in R^n,\ \ A \in R^{n\times n},\ i=0,1,2,3,\ldots\ldots$$

Questions:

Under what condition will the sequence $\{x^{(i)}\}_{i=0}^{i=\infty}$ be convergent ? Prove it and show that the solution is independent on $x^{(0)}$. (17%).

2. Let V and V^* be vector spaces with ordered bases $B = \{b_1\ b_2\ b_3\}$ and $B^* = \{b_1^*\ b_2^*\ b_3^*\}$, respectively. Let $T: V \to V^*$ with $T(b_1) = b_1^* + 2b_2^* - 3b_3^*$, $T(b_2) = 2b_1^* + 2b_2^* - 3b_3^*$ and $T(b_3) = 3b_1^* + b_2^* - 3b_3^*$.

Questions:

(a) Whether T is a linear transformation or not ? Prove it.

(b) If T is a linear transformation, then $T^{-1}(b_1^*)$=?. (20%)

3. Let $\{u_1\ u_2 \ldots\ldots u_n\}$ be an orthonormal basis for R^n and let λ_i be scalars, i=1~n..

Define $A = \lambda_1 u_1 u_1^T + \bullet\bullet\bullet + \lambda_n u_n u_n^T$.

Questions:

What are the eigenvalues and eigenvectors of A ? Prove it. (13%)

4. (1) Consider a graph G which consists of two cycles of length n joined together by a single additional edge. How many possible Euler paths or cycles in G?(3%)

(2) Consider a complete graph with n vertices numbered 0,1,..., n-1. Suppose the cost of the edge (u,v) that connects vertex u and vertex v is | u-v |. Describe a minimum spanning tree for this graph, for general n. (3%)

(3) Simplify the Boolean expression: f(x,y,z) = yz' + xz + y'z + x'yz (3%)

(4) (true or false) If G is a planar graph, and if any number of loops, degree 2 vertices, are added to G, then G remains planar.(3%)

(5)(multiple choices) Which of the following are not in general true for regular sets.

(a) (A+B)* = A* + B*   (b) (AB)* = A*B*   (c) AB + B = AB (3%)

(背面仍有題目,請繼續作答)

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5. (1) Suppose the vertices of a certain binary tree are listed in preorder as A, B, D, F, E, C and in post order as F, D, E, B, C, A. Draw the tree. (5%)

(2) Find a good move from the Nim-position [8,3,7,11]. (Hint: Nim is an ancient game. It is played with several piles of checkers. In this case, there are four piles. The player must remove a positive number of checkers (possibly all) from any one pile of his or her choosing. Play alternates, and the one who picks up the last checker wins.)(5%)

(3) Here the alphabet is {a,b}. Find either an non-deterministic finite automaton (NFA) or a deterministic automaton (DFA) whose language is $a + b^* + (ab)^*$ (5%)

(4) Find a generating function for the sequence $3 + 4^n$. (5%)

(5) Given the inequality $a_n \geq 2a_{n-1}$ $(n \geq 1)$. If $a_0 \geq 5$, show that $a_n \geq 5*2^n$. (5%)

6. Let the universe **U** consist of 8 vertices, (x_i,y_i,z_i), of a cube, whose coordinates are (0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,0,1), (0,1,1), (1,1,0) and (1,1,1). Define the relation **R** on **U** by the condition that $(x_1,y_1,z_1)\mathbf{R}(x_2,y_2,z_2)$ if and only if $x_1 \leq x_2$, $y_1 \leq y_2$, and $z_1 \leq z_2$. By convention, we also agree that $(x_i,y_i,z_i)\mathbf{R}(x_i,y_i,z_i)$ is true. Show that the relation **R** is a partial ordering. (10%)