1. Let $u_1(x)$ and $u_2(x)$ be two homogeneous solutions of the differential equation

$$\frac{d^2y}{dx^2}+p(x)\frac{dy}{dx}+q(x)y=0.$$

Question:

Solve the differential equation

$$\frac{d^2y}{dx^2}+p(x)\frac{dy}{dx}+q(x)y=r(x)$$

in terms of $u_1(x)$, $u_2(x)$ and $r(x)$. (18%)

2. Using the method of Laplace transform to solve the differential equation

$$\frac{d^4y}{dt^4}+(a^2+b^2)\frac{d^2y}{dt^2}+a^2b^2y=h(t)$$

with initial conditions

$$\frac{d^3y}{dt^3}=\frac{d^2y}{dt^2}=\frac{dy}{dt}=y=0 \quad \text{at} \quad t=0. \text{ (18\%)}$$

3. Try an initial value $x^{(0)}$ to solve the equations of the system

$$x^{(i+1)}=Ax^{(i)}+b,\ x^{(i)},b\in R^n,\ A\in R^{n\times n},\ i=0,1,2,3,\ldots\ldots$$

Question:

Under what condition will the iteration be convergent ? Prove it and show that the solution is independent on $x^{(0)}$. (14%).

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4. Expand $\dfrac{1}{1+z}$ in a Taylor series centered at $-2i$ and determine the radius of convergence. 10%

5. Find a bilinear transformation that maps the circle $|z-i|=1$ onto the circle $|w-1|=2$. 10%

6. Integrate $1/(z-z_0)^m$ (m: a positive integer) counterclockwise around any simple closed path C enclosing the point $z=z_0$. 10%

7. Show major steps how to solve the following boundary value problem (without actually solving it): 20%

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x^2}=0$$

B.C.