1. Let

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 3 & 4 \end{bmatrix}$$

Find bases for $N(A)$, $R(A)$, $N(A^T)$ and $R(A^T)$. (16%)

2. Find the best quadratic least squares fit to the data

x	0	1	2	3
y	3	2	4	4

(16%)

3. The vectors $x_1 = 0.5(1,1,1,-1)^T$ and $x_2 = 0.5(1,1,3,5)^T$ form an orthonormal set in R^4. Extend this set to an orthonormal basis for R^4 by finding an orthonormal basis for the nullspace of

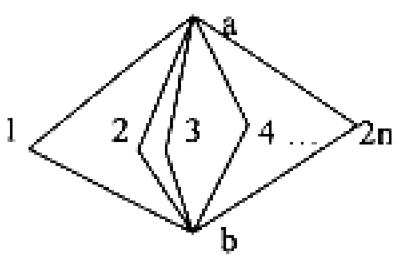
$$\begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & 3 & 5 \end{pmatrix}.$$

(18%)

4 (1)Consider the statement S(n): $n^2 - n + 41$ is prime (i.e., not divisible by any positive integer except itself and 1) for n = 0, 1, 2. Prove it or disprove it by showing an example. (3%)

(2)Some of the following regular expressions are equivalent. Please choose them. (a) $(00)^*(\varepsilon + 0)$, where ε represents an empty string. (b) $(00)^*$ (c) 0^* (d) $(000)^*$ (2%)

(3)The accompanying figure shows a generalization of the graph. How many different Euler cycles does it have? (5%)



(4)True or false (5%)

(a) ( ) Adding an edge to a tree creates exactly one cycle.

(b) ( ) A spanning tree for a graph with n vertices has those n vertices and n-1 edges.

(c) ( ) If a graph G has one more vertex than edge, then it is a tree.

(d) ( ) If G is connected, then it has no cycles.

(e) ( )If G is connected "edge minimally," i.e., removing an edge from G disconnects it, then G has no cycles.

(背面仍有題目,請繼續作答)

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(5) Given the adjacency matrix, determine whether or not the given graph is planar (5%)

	A	B	C	D	E	F	G	H	I	J
A	0	1	0	0	1	1	0	0	0	0
B	1	0	1	0	0	0	1	0	0	0
C	0	1	0	1	0	0	0	1	0	0
D	0	0	1	0	1	0	0	0	1	0
E	1	0	0	1	0	0	0	0	0	1
F	1	0	0	0	0	0	0	1	1	0
G	0	1	0	0	0	0	0	0	1	1
H	0	0	1	0	0	1	0	0	0	1
I	0	0	0	1	0	1	1	0	0	0
J	0	0	0	0	1	0	1	1	0	0

5.(1)A 'delay line' is a circuit for which the output sequence is a delayed version of the input sequence. For example, a two-unit delay line has $y_n = x_{n-2}$ for n=1,2,3,... Design a two-unit delay line Finite State Machine, with input and output alphabet {0,1}. (5%)

(2) Is it possible to design an finite state machine that *divides by 2*? That is, given an input sequence where there are *n* consecutive 1s and *n* is an even number, the output will have *n/2* 1s. For example, 11111111000000...(there are eight 1s in the string) divides by 2 is 1111000...(there are four 1s in the string.). Explain your answer. (5%)

6. Consider the sequence 0, 1, 1/2, 3/4, 5/8, 11/16,...., in which each term is the average of the previous two terms; e.g. the next term will be (1/2) * [(5/8)+(11/16)] = 21/32. Find a formula for the general term. (10%)

7. If P and Q are two implicants of a given Boolean function, let us say that P *is covered by* Q and write $P \leq Q$, if every minterm involved in P is also involved in Q. For example, $x_1' x_2' \leq x_1'$, $x_2x_3 \leq x_2$, etc. Show that the relation of covering is a partial ordering. (10%)