1. 15%

(a) Prove the Laplace Convolution Theorem: If $L\{f(t)\}=F(s)$ and $L\{g(t)\}=G(s)$ both exist for s>c, then $L\{\int_0^t f(\tau)g(t-\tau)d\tau\}=F(s)G(s)$, or equivalently,

$$L^{-1}\{F(s)G(s)\}=\int_0^t f(\tau)g(t-\tau)d\tau. \quad (10\%)$$

(b) Using (a), find $L^{-1}\{\frac{3}{s^2+3s-10}\}=?$ (5%)

2. 15%

(a) What is the Gibbs phenomenon in a Fourier series expansion? (5%)

(b) Expand $f(x)=x^2,\ 0<x<2\pi$ if its period is 2π. (5%)

(c) Prove that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\dots=\frac{\pi^2}{6}$ (5%)

3. 20%

(a) Briefly describe the Residue Theorem. (8%)

(b) Use the residue theorem to evaluate the following integral counterclockwise around any simple closed path such that (1) 0 and 1 are inside C, (2) 0 is inside, 1 is outside, (3)0 is outside, 1 is inside, (4) 0 and 1 are outside. $\oint_C \frac{4-3z}{z^2-z}dz$,

(12%)

4. (a) State the Divergence theorem of Gauss

(b) Use the Divergence theorem to evaluate the following surface integral

$$I=\iint_s (x^3dydz+x^2ydzdx+x^2zdxdy)$$

where s is the closed surface consisting of the cylinder $x^2+y^2=a^2$ $(0\le z\le b)$ with the circular disks at $z=0$ and $z=b$ $(x^2+y^2\le a^2)$ ---14%

5. Solve $y\frac{d^2y}{dx^2}-(\frac{dy}{dx})^2-y^2\frac{dy}{dx}=0$ ---14%

6. Solve $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ with $u_x(0,t)=0$, $u_x(L,t)=0$, and $u(x,0)=f(x)$ ---22%