(1) 20%

We wish to solve f(x)=0 by using the Newton's iteration.

(a) Derive the expression for the iteration process.(5%)

(b) Analyze the iteration error and derive its expression for the method to converge.(5%)

(c) Use the Newton's method to find the roots of the following function:

$f(x) = (x+1)^3(x-1) = 0$, starting with x=1.1 (do three iterations).(5%)

What starting values cause the Newton's method to fail? (2%).

Any modifications can be made to avoid the failure? (3%)

(2) 20%

We wish to solve the roots of the following system of nonlinear equations, using the Newton's iteration method,

$$\begin{cases} xyz - x^2 + y = 1.34 \\ xy - z^2 = 0.09 \\ e^x - e^y + z = 0.41 \end{cases}$$

(a) Derive the expression for the iteration process.(15%)

(b) use the result of part (a) to solve the above equations with the initial guess x=1, y=1 and z=1. (Do two iterations only).(5%)

(3) 20%

(a) Consider the linear system $\begin{cases} 2x + 6y = 8 \\ 2x + 6.00001y = 8.00001 \end{cases}$

Is this problem well-conditioned or ill-conditioned? why? (5%)

(b) Consider $\begin{cases} 2x + 6y = 8 \\ 2x + 5.99999y = 8.00002 \end{cases}$

Solve this system exactly. What does the solution tell you about your answer to part (a)? (5%)

(c) Give a geometric interpretation of your answers to parts (a) and (b). (10%)

(4) 20%

Use the LU decomposition method to solve the following equations L=?, U=? X=?

$$3x_1 - x_2 + x_3 = 1$$
$$3x_1 + 6x_2 - 2x_3 = 0$$
$$3x_1 + 3x_2 + 7x_3 = 4$$

(5) 20%

We want to integrate $\int_a^b f(x)dx$ numerically by the two-term Gaussian quadrature i.e. $\int_a^b f(x)dx = c_1 f(t_1) + c_2 f(t_2)$. Find c_1, c_2, t_1 and t_2. (15%) Use the above result to evaluate $\int_0^{\frac{\pi}{2}} \cos x dx$ and compare the exact result. (5%)