

1. What is the value of

$$\int_C \frac{z+4}{z^2+2z+5} dz$$

- (a) If C is the circle $|z|=1$? (5%)
(b) If C is the circle $|z+1-i|=2$? (5%)
(c) If C is the circle $|z+1+i|=2$? (5%)

2. For the matrix A

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Find two nonsingular matrices P and Q such that PAQ is a diagonal matrix. (15%)

3. For the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}, \quad 0 \leq x \leq L, \quad 0 \leq t, \quad u(0,t) = u(L,t) = 0, \quad u(x,0) = 1,$$

$$\frac{\partial u}{\partial t}(x,0) = 3.$$

Find the solution $u(x,t)$. (20%)

4. The equation $y' + p(x)y = q(x)y^n$ is called Bernoulli equation.

- (1) Shown that by transforming the dependent variable from $y(x)$ to $v(x)$ according to $v = y^{1-n}$ (for $n \neq 0, 1$), the equation can be converted to $v' + (1+n)p(x)v = (1-n)q(x)$. (6%)
(2) $xy' - 2y = x^3y^2$. Find the general solution of the equation. (6%)

5. Obtain the general solutions of the following problems using the method of undetermined coefficients. (12%)

(i) $y'' + y' = 4xe^x + 3\sin x$ (ii) $y'' - 2y' + y = x^2e^x$

6. Use the Laplace transform to find the particular solutions of the following problems. (12%)

(i) $3x' + x = 6e^{2t}$; $x(0) = 0$

(ii) $x'' - 3x' + 2x = 0$; $x(0) = 3, x'(0) = 1$

7. Work out the Fourier series of $f(x)$, given over one period as follows. (14%)

(i) $f(x) = x$ on $(-\pi, \pi)$ (ii) $f(x) = |\sin x|$ (for all x)