(1) 20%

Briefly describe and write down the mathematical expressions for the following numerical methods in solving a system of equations Ax=b, A is a *n x n* matrix, x and b are *n x 1* column vectors.

(a) The Gaussian elimination method (5%)
(b) The Gauss-Seidel iteration method (5%)
(c) The Jacobi iteration method (5%)
(d) The successive over relaxation method (5%)

(2) 20%

Use the LU decomposition method to solve the following equations L=?, U=? X=?

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(3) 20%

We wish to solve the roots of the following three nonlinear equations, using the Newton's iteration method,

$$\begin{cases} f(x,y,z)=0 \\ g(x,y,z)=0 \\ h(x,y,z)=0 \end{cases}$$

Please derive the expression for the iteration process and explain your solution procedure.

(4) 21%

Define the following: (3 points each)

(a) Round-off error, (b) Truncation error, (c) Singular matrix, (d) Rank (of a matrix) (e) Norm (of a matrix), (f) Condition number (of a matrix), (g) residual.

(5) 19%

If we solve the linear equation $\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0$, u = constant > 0. by the finite-difference method as $\frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} + u\frac{\phi_i^n - \phi_{i-1}^n}{\Delta x} = 0$. Show that the method will render the difference equation back to the differential equation as $\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = \alpha\frac{\partial^2 \phi}{\partial x^2} + O(\Delta x^2, \Delta t^2)$. Find $\alpha = ?$, Explain the physical meaning of the extra term.