1. (1) Let **R** be a transitive and reflexive relation on A. Let $\mathbf{R}_1$ be a relation on A such that (a,b) is in $\mathbf{R}_1$ if and only if both (a,b) and (b,a) are in **R**. Is $\mathbf{R}_1$ an equivalence relation? Prove your claim. (5%)

   (2) Let $(A, \leq)$ be a partially ordered set. Let $\mathbf{R}_2$ be a binary relation on A such that for a and b in A, $a\ \mathbf{R}_2\ b$ if and only if $b \leq a$. Is $\mathbf{R}_2$ a partial ordering relation? Prove your claim. (5%)

   (3) What can be said about a relation R that is both a partial ordering and an equivalence relation? Give an example of such a relation on the set {1,2,3}. (2%)

   (4) True or False. Let **R** be an equivalence relation on the set A, and let A_i be one of the equivalent classes. A_i can be the empty set. (2%)

2. (1)Simplify the Boolean expression:

   $$F(w,x,y,z) = wxy'z + xy'z' + wx'z' + w'x'yz' + x'y'z' \quad (5\%)$$

   (2) Find a regular expression for the language of all strings over {x,y} which begin and end with the same character. (5%)

   (3) Draw a diagram of a non-deterministic finite automata which accepts all binary sequences ending with either 1010 or 001, over the alphabet {0,1}. (5%)

   (4) True or False. If we have a finite automaton, either deterministic or non-deterministic, which recognizes a certain set of strings, we can use that automaton to produce a computer program capable of recognizing the same set of strings. (2%)

   (5)True or False. Every language that accepted by a deterministic finite automaton is regular. (2%)

   (6)True or False. It is impossible to construct a finite state machine that takes as input a string of 1s and produce as output a string of 1s twice as long. (2%)

3. (1) True or False. There is a tree with 3 vertices of degree 3, 1 vertex of degree 2 and 3 vertices of degree 1. (2%)

   (2) Let $\mathbf{T}_1$ and $\mathbf{T}_2$ be two trees. Let v_1 and v_2 be two distinct vertices in $\mathbf{T}_1$. Let v_3 and v_4 be two distinct vertices in $\mathbf{T}_2$. Suppose **E** is a graph constructed from $\mathbf{T}_1$ and $\mathbf{T}_2$ by connecting v_1 with v_3, and v_2 with v_4.

   (a) Is **E** a tree? Prove your claim. (5%)

   (b) Suppose that it is required that **E** has an Euler cycle (Eulerian circuit). Will it be possible? If it is possible, please describe your approach. If it is impossible, show the reason. (5%)

   (3)True or False. If **G** is a nonplanar graph, then any connected subgraph of **G** is also nonplanar. (2%)

(背面仍有題目,請繼續作答)

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4. (1)Suppose that there is a row of n 0s and 1s. It is required to rearrange them so that the 0s will be grouped at the right and the 1s will be grouped at the left.
   (a) Design an algorithm, (5%)
   (b) show that it is correct, and (5%)
   (c) determine its complexity. (5%)
   (d) Give an example of the algorithm. (2%)

   (2) True or false. The time complexity of algorithm A is $O(n^2)$, and that of algorithm B is $O(n^2 \ln r)$. We conclude that algorithm A is superior to algorithm B. (2%)

   (3) True or false. An O(50n) algorithm is also an O(3n) algorithm. (2%)

5. (1)Find a simple expression for the generating function of each of the following infinite sequence (or discrete numeric function): 1, -2, 3, -4, 5, -6,... (5%)
   (2)Convert the generating function: $A(z) = (z^5)/(5-6z + z^2)$ into an explicit expression for F_n. (5%)

6. (1)Let k be a positive integer. The Cartesian k-space denoted by R^k, is the set of all sequences $(a_1, a_2, \ldots, a_k)$ of k real numbers. Which of the following sets of vectors in R^3 are linearly dependent and which are linearly dependent? (5%)
   (a) E = {(1,1,1), (0,1,0),(1,0,1)}
   (b) F = {(1,1,1), (1,1,0), (1,0,0)}
   (c) G = {(1,1,1), (1,1,0), (1,0,1),
   (d) H = {(1,0,0), (0,1,0), (1,1,1)}
   (e) K = {(1,1,1), (0,1,0), (0,0,1)}

   (2) The vectors in $P_n(R)$ are polynomials: $P_m(x) = a_0 + a_1x + \ldots + a_mx^m$ of degree less than or equal to n, that is, $m \le n$. Which of the following sets of vectors in $P_2(R)$ are linearly dependent? (5%)
   (a) E = { $1, x, x^2$ },
   (b) F = { $1+x, 1-x, x^2, 1$ },
   (c) G= { $x^2-1, x+1, x^2-x, x^2+x$ }
   (d) H = { $x-x^2, x^2-x$ },
   (e) K = { $1, 1-x, 1-x^2$ }

   (3) Show the polynomials: $A_1 = 1, A_2 = t-1, \quad A_3 = (t-1)^2$ form a basis for $P_2(R)$. Find the coordinates of the vector: $B = 2t^2 - 5t + 6$ relative to this ordered basis. (10%)