1. The partial differential equation and the corresponding initial conditions are given as

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2},\ t \geq 0,\ -\infty < x < \infty,\ u(x,0) = f(x),\ \left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x).$$

Find $u(x,t)$. (20%)

2. Find the surface area of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ that is above the $xy-$plane and within the cylinder $x^2 + y^2 = b^2$, where $0 < b < a$. (15%)

3. Solve the equation $X^2 - 5X + 3I = \begin{bmatrix} 1 & -4 \\ 2 & -5 \end{bmatrix}$. (15%)

4. Solve the equation $y''' - 6y'' + 11y' - 6y = 6$ with $y(0) = 0,\ y'(0) = 1,\ y''(0) = 1$. (12%)

5. Find the Fourier series of f(x) on the given interval. (12%)

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$$

6. Use the Laplace transform to find the particular solutions of the following problem. (12%)

$y'' - 6y' + 9y = t^2 e^{3t};\quad y(0) = 2,\ y'(0) = 6$

7. Solve the following system. (14%)

$x' = 3x - y - 1$

$y' = x + y + 4e^t$