(1) 10%

(a) A binary system (二進位) is usually used in a computer, please convert the following binary expression $(1101.001)_2$ into the decimal (十進位) expression. (5%)

(b) The decimal (十進位) expression $(20.125)_{10}$, what is its binary expression? (5%)

(2) 30%

Let $f(x)=x^2-6=0$, With $x_0=3$ and $x_1=2$, find x_3=?

(a) Use the Secant method. (10%) (b)Use the Newton's method. (10%) (c) Which of (a) or (b) is closer to $\sqrt{6}$? Why? (10%)

(3) 20%

(a) Write down the Lagrange interpolation polynomial passing the following three given points: $(x_1,y_1)=(-2,4),(x_2,y_2)=(0,2),(x_3,y_3)=(2,8)$ (5%)

(b) If we use divided difference to obtain interpolation polynomial passing the above three points, what is the polynomial? (5%)

(c) If we use $y=ax^2+bx+c$ to fit the above three points, what are a, b, c? (5%)

(d) Would the resulting polynomials in (a) , (b)and (c) be different? Why? Explain your answer.(5%

(4) 20%

We can use cubic spline to do interpolation for a set of data (x_i, y_i) given as (0,1.), (0.1,1.105), (0.2,1.221), (0.3,1.350) .

The interval spacing is constant , i.e. $x_i - x_{i-1} = h = 0.1$. It can be shown that the cubic spline relations is: $hs_{i-1}+4hs_i+hs_{i+1}=6h^{-1}(y_{i+1}-2y_i+y_{i-1})$, where s_i is the second derivative of the curve at point i. If we use natural cubic spline, ie, $s_0=s_3=0$. Find the interpolation values at x=0.05, x=0.15, and x=0.25 The spline curves are given as:

$$\begin{cases} g_i(x)=a_i(x-x_i)^3+b_i(x-x_i)^2+c_i(x-x_i)+d_i \\ a_i=(s_{i+1}-s_i)/(6h) \\ b_i=0.5s_i \\ c_i=(y_{i+1}-y_i)/h-(2hs_i+hs_{i+1})/6 \\ d_i=y_i \end{cases}$$

(5) 20%

We want to integrate the initial-value problem $\frac{dy}{dx}=f(x,y),\ y(x_0)=y_0$ by the Runge-Kutta method (local error is 3th order, global error is 2rd order in step-size= h):

$$y_{n+1}=y_n+ak_1+bk_2$$
$$k_1=hf(x_n,y_n)$$
$$k_2=hf(x_n+\alpha h, y_n+\beta k_1)$$

Prove that

$$a+b=1,\ \alpha b=1/2,\ \beta b=1/2$$