1. Determine whether each statement in the following true or false. If the statement is false, give a counterexample. If the statement is true, briefly prove or explain your concept.

(1) Suppose that R is an relation on a set X that is symmetric and transitive but not reflexive. Suppose also that $|X| \geq 2$. ($|X|$: number of elements in X.) Define the relation R' on X by $R' = X \times X - R$. (i) R' is reflexive. (ii) R' is symmetric. (iii) R' is not antisymmetric. (iv) R' is transitive. (v) R' is a partial order. (5%)

(2)(i) Assume that the functions f, g take on only positive integers If $f(n) = O(g(n))$, then $g(n) = O(f(n))$. ($O(f(n))$ is the *big-oh notation* for f)

(ii) Assume that the functions f, g take on only positive integers If $f(n) = \theta(g(n))$, then $g(n) = \theta(f(n))$. ($\theta(f(n))$is the *theta notation* for f)

(iii) $n! = O(n^n)$.

(iv) $2^n = O(n!)$.

(v) $\lg n! = O(n \lg n)$. ($\lg x$: logarithm to the base 2 of x) (10%)

(3) (i) Any tree with two or more vertices has a vertex of degree 1.

(ii) A tree is a bipartite graph.

(iii) T is a tree if and only if T is connected and when an edge is added between any two vertices, exactly one cycle is created.

(iv) If T is a tree with six vertices, T must have five edges.

(v) An acyclic graph with eight vertices has seven edges. (10%)

2. A robot can take steps of 1 meter or 2 meters. Find the number of ways the robot can walk n meters. That is, to find a recurrence relation for the sequence and solve the recurrence relation. (10%)

3. (1) A shipment of 100 compact discs contains five defective discs. In how many ways can we select a set of four compact discs that contains more defective than nondefective discs? (5%)

(2) Trisha, Roosevelt, and Joe write programs that schedule tasks for manufacturing toys. The following table shows the percentage of code written by each person and the percentage of buggy code for each person.

	Trisha	Roosevelt	Joe
Percent of code	30	45	25
Percent of bugs	3	2	5

Given that a bug was found, find the probability that it was in the code written by Joe. (5%)

4. (1) Give an example of a graph that has an Euler cycle but contains no Hamilton cycle. (5%)

(2) Consider a topology of an $m \times n$ matrix, for which values of m and n does the graph contains an Euler cycle? (5%)

(3) Let A be the adjacency matrix of a graph G with n vertices. Let $Y = A + A^2 + \ldots + A^{n-1}$. If some off-diagonal entry in the matrix Y is zero, what can you say about the graph G? (5%)

5. (1) Prim's algorithm is an example of a greedy algorithm. A greedy algorithm is an algorithm that optimizes the choice at each iteration without regard to previous choices. The principle can be summarized as "doing the best locally." Give an example of the use of greedy method that does not lead to an optimal algorithm. (5%)

(2) Mr. Someone claims to have discovered an algorithm that uses at most $100n$ comparisons in the worst case to sort n items, for all $n \geq 1$. His algorithm repeatedly compares two elements and, based on the result of the comparison, modifies the original list. Give an argument that shows that he must be mistaken. (5%)

6. (1) Design a half-adder using only NAND gates. (5%)

(2) Assume the input is always a bit string. Design a finite-state machine that outputs 1 if an even number of 1's has been input; otherwise, outputs 0. (5%)

(3) Design nondeterministic finite-state automata that accept the strings over $\{a,b\}$ that containing bab and bb. (5%)

7. Use induction to show that n straight lines in the plane divide the plane into $(n^2+n+2)/2$ regions. Assume that no two lines are parallel and that no three lines have a common point. (10%)

8. If R is an equivalence relation on a set X, and $|X| = |R|$, what is the relation R? (5%)