1. (20%) An approximate model for steering a large ship moving forward at constant velocity is given by

$$\frac{dV}{dt} = -0.8V - 1.4R - 0.5V|V| + 0.1u$$

$$\frac{dR}{dt} = -V - R + u$$

where V is the velocity, R is the rate of rotation and u is the rudder angle.

(a) (5%) Determine the all equilibria when the rudder angle is zero.
(b) (5%) Determine the linearization of the system about the equilibria.
(c) (5%) Determine the stability of the equilibria.
(d) (5%) How will the ship behave if the rudder angle is zero.

2. (20%) Consider the *discrete time* system given by

$$x(k+1) = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(k)$$

(a) (5%) Is the system controllable?
(b) (5%) If the initial condition is $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, find the control sequence $u(0)$, $u(1)$ required to drive the state to the origin in two sample periods.
(c) (5%) Is the system observable?
(d) (5%) Given the observation sequence $y(1)$, $y(2)$, and $u \equiv 0$, find the initial state $x(0)$.

3. (20%) Consider the system given by

$$\frac{dx}{dt} = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t).$$

(a) (5%) Assume that all states are measured. Please find a state feedback of the form $u = -K_x x + K_r r$ so that the closed-loop system from reference input r to output y has the characteristic polynomial $s^2 + s + 1$.
(b) (5%) Assume that only the output y is measured. It is desired to have an observer for the state of the system which has the characteristic polynomial $s^2 + 2s + 4$. Please give the equation for the observer and the values of the observer gains.
(c) (5%) By using the observer-based state feedback control, please write the equations of the closed-loop system in terms of the original state x and the observer error $\tilde{x}$.
(d) (5%) Find the transfer function from reference input r to output y for the closed-loop system.

4. (20%)

(a) (10%) For a negative unity feedback system, the open-loop transfer function is $G(s) = \dfrac{K}{s(s+2)}$. Please specify the gain K so that the output has an overshoot of no more than 10% in response to a unit step input.
(b) (10%) For a negative unity feedback control system, the transfer function of controlled system is $G(s) = \dfrac{1}{s(s+3)}$ and transfer function of controller is $C(s) = \dfrac{K(s+z)}{(s+p)}$. Please find K, z, and p so that the closed-loop system has a 10% overshoot to a step input and a settling time of 1.5 sec (1% criterion).

5. (20%) The z-transform of a signal is defined as

$$Z\{x(t)\} = \sum_{k=0}^{\infty} x(kT) z^{-k}.$$

Please find

(a) (10%) $Z\{0.5t + \exp^{-2t}\} = ?$
(b) (10%) $Z\{\cos(10t)\} = ?$

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