

1. The following data is listed as

x	1	2	3	4
y	3	5	6	2

Questions:

- (a) Set up the Lagrange polynomial such that $y(x_i) = y_i$. (5%)
- (b) Further, what are the values of a and b in $y = ax + b$ to least square approximate the data listed in the table. (5%)
2. A sphere is defined by $x^2 + y^2 + z^2 = a^2$. Calculate the surface area of sphere intersected by the cylinder $0 \leq x^2 + y^2 \leq b^2$ and $z \geq 0$, where $b < a$. (15%)
3. What is the solution $y(t)$ for the problem $y'' + (a+b)y + aby = f(t)$, $y(0) = c$ and $y'(0) = d$. (15%)
4. Evaluate the integral
- $$\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx \quad a, b \text{ positive. (15\%)}$$

5. The equation is given as

$$X^2 - 4X + 4I = A, \quad A = \begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}$$

Questions:

- (a) What are eigenvalues and the corresponding eigenvectors of A ? (10%)
- (b) What are the solutions of X ? (15%)

(背面仍有題目,請繼續作答)

6. Suppose $U_1(x, y)$ to be the solution of the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \leq x, y \leq a,$$

with the boundary conditions

$$u(x, 0) = g(x), \quad u(x, a) = 0, \quad u(0, y) = 0, \quad u(a, y) = 0.$$

Questions:

- (a) What is the solution $U_2(x, y)$ in terms of $U_1(x, y)$ for these boundary conditions $u(x, 0) = 0, u(x, a) = g(x), u(0, y) = 0, u(a, y) = 0$? (5%)
- (b) What is the solution $U_3(x, y)$ in terms of $U_1(x, y)$ for these boundary conditions $u(x, 0) = 0, u(x, a) = 0, u(0, y) = g(y), u(a, y) = 0$? (5%)
- (c) What is the solution $U_4(x, y)$ in terms of $U_1(x, y)$ for these boundary conditions $u(x, 0) = 0, u(x, a) = 0, u(0, y) = 0, u(a, y) = g(y)$? (5%)
- (d) What is the solution $U_5(x, y)$ in terms of $U_1(x, y)$ for these boundary conditions $u(x, 0) = g(x), u(x, a) = 3g(x), u(0, y) = 2g(y), u(a, y) = 4g(y)$? (5%)