允許考生攜帶特殊功能計算機

(1) 30%

Given a set of data points:

$(x_1, y_1) = (-1, 0.3679)$, $(x_2, y_2) = (0, 1)$, $(x_3, y_3) = (1, 2.7183)$

(a) Write down the Lagrange interpolation polynomial passing the above points (5%)

(b) If we use divided difference to obtain interpolation polynomial passing the above three points, what is the polynomial? (5%)

(c) If we use $y = ax^2 + bx + c$ to fit the above three points, what are a, b, c? (5%)

(d) Would the resulting polynomials in (a), (b)and (c) be different? Why? Explain your answer. (5%).

(e) What is the expression if we use the least squares polynomial of degree 2? (10%)

(2) 20%

Given Chebyshev polynomials as: $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$, $T_0(x) = 1$, $T_1(x) = x$.

we want to approximate $y(x) = e^{-x}$ using $y(x) \approx \sum_{i=0}^{3} a_i T_i(x)$ in the interval $[-1,1]$.

What are the coefficients $a_i = ?$ (20%)

(3) 20%

We wish to solve the roots of f(x,y)=0 and g(x,y)=0,two nonlinear equations, using the Newton's iteration method.

(a) Derive the expression for the iteration process.(15%)

(b) use the result of part (a) to solve

$\begin{cases} x^2 + y^2 + xy = 28 \\ xy^2 = 16 \end{cases}$ with the initial quess x=1 and y=1. (Do two iterations only).(5%)

(4) 20%

Use any method you like to prove the following relations: (h is the equal space distance between f_i).

(a) $f'(x_0) = \dfrac{-f_2 + 4f_1 - 3f_0}{2h} + O(h^2)$ (5%)

(b) $f''(x_0) = \dfrac{-f_3 + 4f_2 - 5f_1 + 2f_0}{h^2} + O(h^2)$ (5%)

(c)For non-equal spacing points, f(x-s), f(x), f(x+t), find $f'(x) = ?$ (10%)

(5) 10%

Define and explain the followings:

(a) Round-off error. (b) Truncation error. (c) Gaussian elimination method.

(d) Gauss-Seidel Iteration method