1. (20%)

(1) Let R and S be relations on X. Determine whether each statement in (i)-(iv) is true or false. If the statement is false, give a counterexample.

( )(i) If R and S are transitive, then $R \cap S$ is transitive.

( )(ii) If R and S are reflexive, then $R \cup S$ is reflexive.

( )(iii) If R and S are symmetric, then $R \cap S$ is symmetric.

( )(iv) If R and S are antisymmetric, then $R \cup S$ is antisymmetric.

(2) Assume that the functions f,g and h take on only positive values. The big oh notation for f is referred as $f(n) = O(g(n))$. The omega notiaion of f is $f(n) = \Omega(g(n))$. The theta notation for f is $f(n) = \Theta(g(n))$. Determine whether each statement in (v)-(vii) is true or false. If the statement is false, give a counterexample.

( )(v) If $f(n) = \Theta(h(n))$ and $g(n) = \Theta(h(n))$, then $f(n) + g(n) = \Theta(h(n))$

( )(vi) If $f(n) = \Theta(g(n))$, then $g(n) = \Theta(f(n))$.

( ) (vii) If $f(n) = O(g(n))$, then $g(n) = \Omega(f(n))$

(3) Answer true of false for (viii) to (x).

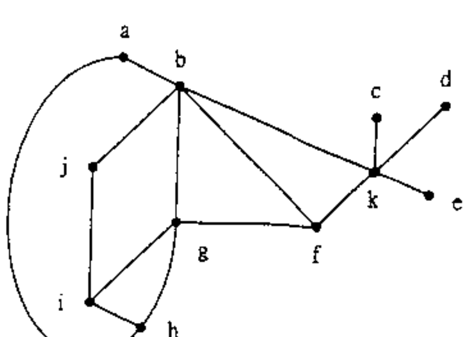
( )(viii) If T_1 and T_2 are isomorphic as rooted trees, then T_1 and T_2 are isomorphic as (free) trees.

( )(ix) If T_1 and T_2 are rooted trees that are isomorphic as free trees, then T_1 and T_2 are isomorphic as rooted trees.

( )(x)If T is a rooted tree with six vertices, the height of T is at most 5.

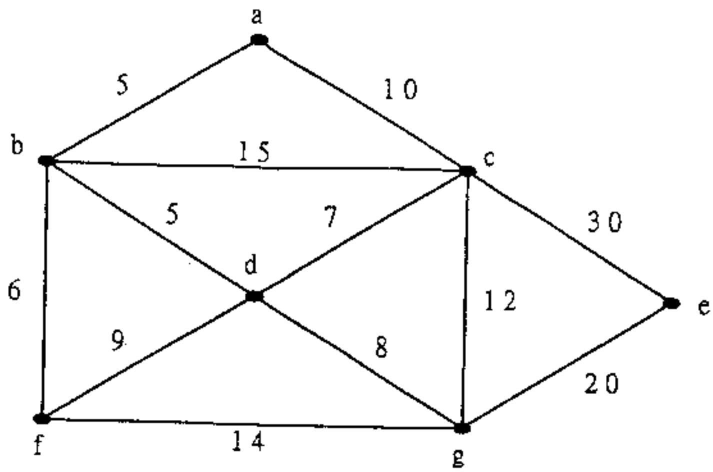
2. (1)When does the complete graph K_n contain an Euler cycle?(5%)

(2)Refer to the following graph. The vertices represent a computer. An edge connects two computers if there is a communication link between the two. Notice that any computer can communicate with any other either directly through a communication link or by having others relay the messages. What is the maximum number of links that can be broken with communication among all computers still possible?(5%).



(背面仍有題目,請繼續作答)

(3) In the following graph the vertices represent offices and the numbers on the edges represent the costs of building a dedicated network link. Find a least-expensive link system that connects all the offices.(5%)



(4) Let G be a connected graph. The distance between vertices v and w in G, dist(v,w), is the length of a shortest path from v to w. The diameter of G is defined as: d(G) = max{dist(v,w)| v and w are vertices in G}. Find the diameter of the graph above, i.e. the graph in problem 2.(3).(5%)

3. (1)When a connected, weighted graph and vertices a and z are input to the following algorithm, it returns the length of a shortest path from a to z. If the algorithm is correct, prove it; otherwise, give an example of a connected, weighted graph and vertices a and z for which it fails.(10%)

```
procedure algor(w, a, z)
    length :=0
    v :=a
    T:= set of all vertices
    while v is not equal to z do
        begin
            T:= T – {v}
            choose x belongs to T with minimum w(v,x)
            length := length + w(v,x)
            v := x
        end
    return(length)
    end algor.
```

(2) Select a theta notation from among $\Theta(1)$, $\Theta(\lg n)$, $\Theta(n)$, $\Theta(n\lg n)$, $\Theta(n^2)$, $\Theta(n^3)$, $\Theta(2^n)$, $\Theta(n!)$ or $\Theta(\lg\lg n)$ for the number of times the statement x:= x+1 is executed. (Note that you must analysis the complexity of the pseudocode).(10%)

```
i:=2
while i<n do
begin
   i := i * i
   x:=x+1
end
```

4. (1) A sequence S where S_n denotes the number of n-bit strings that do not contain the pattern 00. Find a recurrence relation and initial conditions for the sequence S_n. And solve S_n. (10%)
(2) Let *N={D}*, *T= {d, +, -}*, and *P={D→D-D++D-D, D→d, +→+, - → -}*. We regard *G(N,T,P,D)* as a context-free grammar. Does *d-d++d-d* belongs to the language defined by G? Why or why not?(5%) If we interpret the symbol *d* as a command to draw a straight line of a fixed length in the current direction; we interpret + as a command to turn right by 60°; and we interpret – as a command to turn left by 60°. If we begin at the left and the first move is horizontal to the right, when the string is interpreted. Please define a legal string of the language and draw the corresponding graph for it. (5%)

5. (1) Let **X** = {1, 2, 3,…,10}. Define a relation **R** on **X** x **X** by (a,b)**R**(c,d) if a + d = b + c. Show that **R** is an equivalence relation on **X** x **X**. (10%)
(2) A 2's module is a circuit that accepts as input two bits b and FLAGIN and outputs bits c and FLAGOUT. If FLAGIN = 1, then c=b' and FLAGOUT=1. If FLAGIN=0 and b=1, then FLAGOUT=1. If FLAGIN=0 and b=0, then FLAGOUT=0. If FLAGIN=0 then c=b. Design a circuit to implement a 2's module. (10%)