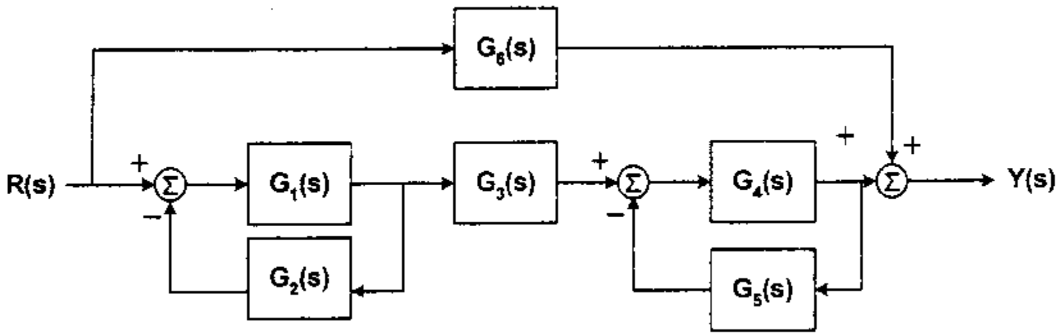
1. (10%) Please find the transfer functions for the following block diagram:



2. (15%) Please find the unit step response and the frequency response, respectively, for the following systems:

(7%) (a) $\dfrac{4}{s+1}$; (8%) (b) $\dfrac{2}{s^2+0.2s+1}$

3. (15%) Consider the lead compensator

$$C_1(s) = K\frac{Ts+1}{\alpha Ts+1} \quad \text{where} \quad \alpha < 1$$

(a)(5%) Show that the phase of the lead compensator is given by

$$\phi = \tan^{-1}(T\omega) - \tan^{-1}(\alpha T\omega)$$

(b) (5%) Show that the frequency where the phase is maximum is given by $\omega_{\max} = \dfrac{1}{T\sqrt{\alpha}}$

and that the maximum phase corresponds to $\phi_{\max} = \sin^{-1}\left(\dfrac{1-\alpha}{1+\alpha}\right)$

(c) (5%) For a compensator $C_2(s) = K'\dfrac{s+z}{s+p}$, please show that the frequency where the phase is maximum is given by $\omega_{\max} = \sqrt{|z||p|}$.

4. (25%) The "inverted-pendulum balancing problem" involves a cart of mass M running a straight horizontal track under the influence of a force u. Friction proportional to the cart's velocity opposes its motion; the friction coefficient is k. On top of the cart is a frictionless hinge connected to a lightweight rod of length l, with a concentrated mass m at its end. Denoting δ for the cart's displacement from the equilibrium and ϕ for the angle between the rod and the vertical yields the following differential equations:

$$(M+m)\ddot{\delta}+ml\cos(\phi)\ddot{\phi}-ml\sin(\phi)\dot{\phi}^2+k\dot{\delta}=u$$
$$l\ddot{\phi}-g\sin(\phi)+\cos(\phi)\ddot{\delta}=0$$

Assume $M=m=k=l=g=1$ for simplicity.

(a) (5%) Find the state-space equation for the system (the state $x=(\delta \quad \dot{\delta} \quad \phi \quad \dot{\phi})$).

(b) (5%) Find a linear system of the form $\dot{x}=Ax+Bu$ (Using the approximation $\sin(\phi)\approx\phi$, $\cos(\phi)\approx 1$ and $\phi\dot{\phi}\approx 0$) for the part (a).

(c) (5%) Find a coordinate-transformation matrix P such that the linear state equation can be expressed by a controllable form in the new coordinate.

(d) (5%) Find a state feedback gain vector K such that all four poles of the closed-loop system equal -1.

(e) (5%) If the output is set as $y=x_3$, then show that the linear system is not observable. Which state variables cannot be distinguished from this output y.

5. (15 %) A control system has the closed-loop transfer function

$$\frac{Y(s)}{R(s)}=\frac{b_0 s+a_2}{s^2+a_1 s+a_2}$$

Please find the parameters (a_1, a_2, b_0) such that the following specifications are satisfied simultaneously:

(i) The rise time $t_r<0.1\,\text{sec}$; *(ii)* The overshoot $M_p<20\%$; *(iii)* The settling time $t_s<0.5\,\text{sec}$; *(iv)* The steady-state error to step reference input is zero; *(v)* The velocity-error constant $K_v\geq 100$.

6. (20%) A negative unity-feedback control system has a proportional controller K and transfer function

$$G(s)=\frac{(s+3)}{(s+1)(s-2)}.$$

(a) (5%) Plot the root-locus for $0\leq K<\infty$.

(b) (5%) Find the breakaway point and reentry point of the root-locus.

(c) (5%) Show that the locus which is not on the real axes will be a circle, and find its center and radius.

(d) (5%) Find the range of K in which the control system is stable.