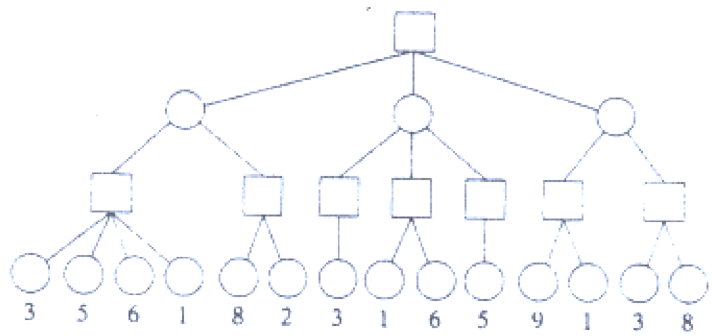
1. (20%)Determine whether each statement in the following is true or false. If the statement is false, give a counter example.
   (1) Let R and S be relations on **X**. If R and S are transitive, then $R \cup S$ is transitive.
   (2) Assume that the functions f, g and h take only positive values, $f(n) + g(n) = \Theta(h(n))$, where $h(n) = min\ \{f(n), g(n)\}$. ($\Theta(h(n))$ is theta notation)
   (3) The set $L = \{\ x_1 \ldots x_n \mid x_1 \ldots x_n = x_n \ldots x_1\}$ of strings over {a,b} is a regular language.
   (4) If the capacity of a cut in a network is equal to C_a, then the value of some flow is greater than or equal to C_a.
   (5) Consider a "shortest-path algorithm" in which at each step we select an available edge having minimum weight incident on the most recently added vertex. This algorithm will always find a shortest path.
2. Answer the following questions briefly.
   (1) Represent the prefix expression – * E / B D – C A as a binary tree. Also write the postfix form and the fully parenthesized infix form of the expression.(10%)
   (2) The n-cube has 2^n vertices, $n \geq 1$. An edge connects two vertices if the binary representation of their labels differs in exactly one bit. The n-cube may also be described recursively. The 1-cube has two vertices, label 0 and 1, and one edge. Let H_1 and H_2 be two (n-1)-cubes whose vertices are labeled in binary $0,\ldots,2^{n-1}-1$. We place an edge between each pair of vertices, one from H_1 and one from H_2, provided that the vertices have identical labels. (a)Draw a 3-cube and find a Hamiltonian cycle in the 3-cube. (b) Prove that the n-cube is bipartite for all $n \geq 1$. (10%).
   (3) If a binary tree of height h has $n \geq 1$ vertices, then $lg\ n < h+1$. (5%)
   (4) Evaluate the root of the tree using depth-first search with alpha-beta pruning. Assume that children are evaluated left to right. For each vertex whose value is computed, write the value in the vertex. Place a check by the root of each subtree that is pruned. The value of each terminal vertex is written under the vertex.(10%)



(背面仍有題目,請繼續作答)

3. (1) Design a circuit that multiplies the binary numbers x_2x_1 and y_2y_1. The output will be of the form $z_4z_3z_2z_1$. (10%)

(2) Explain how to construct a nondeterministic finite-state automaton that accepts the language $L_1L_2 = \{\alpha\beta \mid \alpha \text{ belongs to } L_1, \ \beta \text{ belongs to } L_2\}$(10%)

(3) Given a function defined by the recurrence relations

$A(m,0) = A(m-1), \ m = 1,2,\ldots$

$A(m,n) = A(m-1, A(m,n-1)), \ m=1,2,\ldots; \ n=1,2,\ldots$

And the initial conditions $A(0,n) = n + 1, \ n = 0, 1, \ldots$

Prove that $A(2,n) = 3 + 2n, \ n=0,1,\ldots$(10%)

4. Given an algorithm as follows.

input: a sequence $s_1, s_2, \ldots, s_n$ of zeros and ones

output: $s_1,\ldots,s_n$ where all the zeros precede all the ones

```
procedure sort(s,i,j)
 if i=j then return
 if s_i=1 then
      begin
          Swap(s_i,s_j) //exchange the positions of s_i and s_j
          Call sort(s,i,j-1)
      end
 else
      Call sort(s, i+1,j)
end sort.
```

(1) Prove that *sort* does produce as output a arranged version of the input sequence in which all of the zeros precede all of the ones.(5%)

(2) Let b_n denote the number of times sort is called when the input sequence contains n items. Write a recurrence relation for b_n.(5%)

(3) Solve your recurrence relation of b_n.(5%)