1. Solve $x\dfrac{dy}{dx}+y=0$ by a series method. (10)

2. Given a periodic function with a period of 2 as follows:
   $f(x)=1,\ 0<x<1;\ f(x)=0,\ -1<x<0$. Find the following Fourier series parts: (20)
   a. a_0
   b. a_n
   c. b_n
   d. $f(0)$

3. If $f(x)=\dfrac{d}{dx}\displaystyle\int_x^{3x} e^{y^2}\,dy$, find $f(x)$. (10)

4. Give a general definition of orthogonal functions. Show that Legendre polynomials satisfying $(1-x^2)\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+m(m+1)y=0$ are orthogonal functions. (15)

5. Find the eigenvalues and eigenvectors of $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$. Verify whether the eigenvectors are orthogonal or not. If they are not orthogonal, make them orthogonal. (20)

6. Solve the wave equation $\dfrac{\partial^2 u}{\partial t^2}=c^2\dfrac{\partial^2 u}{\partial x^2}$ with conditions only given as $u(x,0)=x$ and $\dfrac{\partial u(x,0)}{\partial x}=1$. (15)

7. Solve $\dfrac{dy}{dt}+y=\delta(t)$ where $\delta(t)$ is Dirac delta function. (10)