1. (10%) Examine if the following relations belong to equivalence relation, why?
   A. R = {(a, a), (b, c), (c, b), (d, d)} on {a, b, c, d}
   B. R = {(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (5, 5)}

2. (10%) What is the difference between big-O (*O*) and theta (*Θ*) notations for analyzing time complexity? Find a theta notation for the worst-case time required by the following algorithm:

   Void function (int *n*)

   ```
           {
                   int r;
                   r = 1;
                   while (r < n )   {
                           X; r = 2*r; }
           }
   ```

3. (10%) For the following divide-and-conquer algorithm, derive its time complexity in terms of theta (*Θ*) notation.

   T(n) = 2*T(n/2) + n

4. (10%) Let R_n denote the number of regions into which the plane is divided by n lines. Assume that no three lines meet in a point. Derive a recurrence relation for the sequence $R_1, R_2,\dots$

5. (10%) Draw a graph containing an Euler cycle. How can you prove the correctness of your graph?

6. (10%) The function *F(character) = n* denotes the frequency of occurrence of the *character*. For the following characters, find the Huffman code for them. *F(a) = 10, F(b) = 15, F(c) = 20, F(d) = 25, F(e) = 50.*

7. (a) (5%) Draw a graph represented by the following incidence matrix.

   A 1 1 1 0 0 0 0
   B 1 0 0 1 0 1 0
   C 0 0 1 0 1 0 1
   D 0 1 0 1 1 0 0
   E 0 0 0 0 0 1 1

   (b) (10%) Find a spanning tree by using depth-first search for the graph.

（背面仍有題目,請繼續作答）

8. Given a finite state automaton that accepts precisely those strings over {a,b} as follows. (a)Please write an algorithm that determines whether a string is accepted by the finite-state automaton.(10%) (b) Are the string *aabaa, abaa, abba* accepted by the finite-state automaton? (5 %)
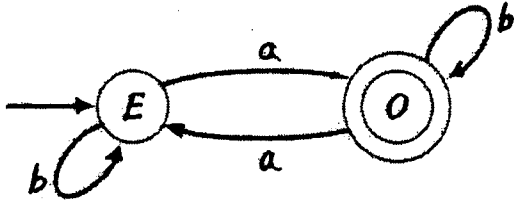
9. (10%) Give the definitions of regular grammar, context-sensitive grammar, and context-free grammar.