1. Use the equation $y = ax^2 + bx + c$ and the least square method to simulate the following data

x	1	2	3	4
y	3	1	4	6

How are these values of a, b and c ? (20%)

2. Solve $\dfrac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} + U(t)\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$, with the initial conditions $\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$.

$U(t)$ is the unit step function.   (20%)

3. Find the surface area of that portion of the sphere $x^2 + y^2 + z^2 = a^2$ that is above the $xy-$plane and within the cylinder $x^2 + y^2 = b^2$, $0 \le b \le a$. (20%)

4. Solve the diffusion equation $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} + e^{-2t}$, $0 \le x \le 1$, $0 \le t$, with the boundary conditions $u(0,t) = u(1,t) = 0$, and the initial condition $u(x,0) = 0$。

(20%)

5. (a) Find the Laurent series expansion of $f(z) = \dfrac{1}{6 - z - z^2}$ in the domain

$2 < |z| < 3$.   (10%)

(b) Calculate $\displaystyle\oint_{|z|=3/2} \dfrac{-3z + 4}{z(z-1)(z-2)}dz$.   (10%)