1. (20%) Consider the circuit shown in **Fig. 1** where the states are defined as follows: x_1 = current through the inductor; x_2 and x_3 = voltage across the capacitors.

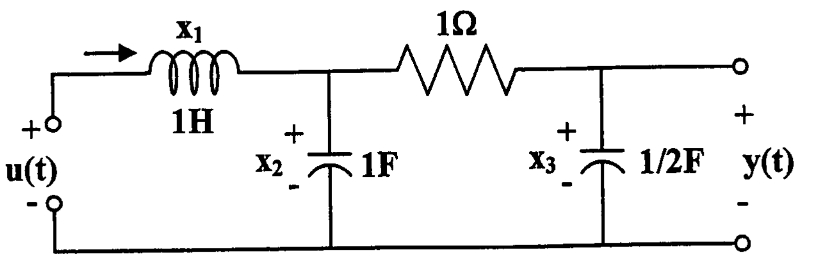

Fig. 1

(a) (5%) Please write down the state space equation and output equation.

(b) (5%) Is the system controllable? Is the system observable? Why?

(c) (5%) What is the system transfer function $G(s)=\dfrac{Y(s)}{U(s)}$?

(d) (5%) Please find the output response at the steady state $y_{ss}(t)$ for $u(t)=2\sin(2t)$

2. (20%) Consider the feedback control system with an open-loop plant $G(s)=\dfrac{\omega_n^2}{s(s+2\varsigma\omega_n)}$ shown in **Fig. 2** where $\omega_n>0$ and $0<\varsigma<1$.

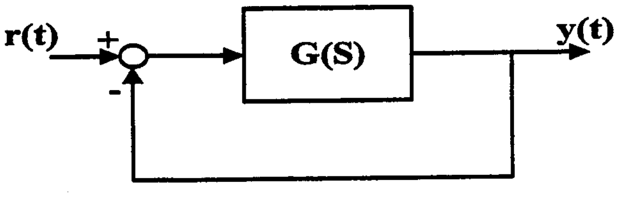

Fig. 2

(a) (10%) Please find the unit step response $y(t)$.

(b) (10%) Please show that the step response has a peak value of

$$y(t^*)=\left(1+\exp^{-\frac{\pi\varsigma}{\sqrt{1-\varsigma^2}}}\right) \text{ at time } t^*=\frac{\pi}{\omega_n\sqrt{1-\varsigma^2}}.$$

3. (20%) (a) (14%) The block diagram of a control system with two feedback loops is shown in **Fig. 3**.

(i) (7%) Please construct the root locus of the system for $0<K_1<\infty$ when $K_2=0$.

(背面仍有題目,請繼續作答)

(ii) (7%) Please construct the root locus of the system for $0 < K_2 < \infty$ when $K_1 = 10$.

(b) (6%) Consider the characteristic equation of a closed-loop system as follows:

$$s^3 + 3K s^2 + (K+1)s + 6 = 0$$

Please find the value of K for which the system is stable using the Routh-Hurwitz criterion.

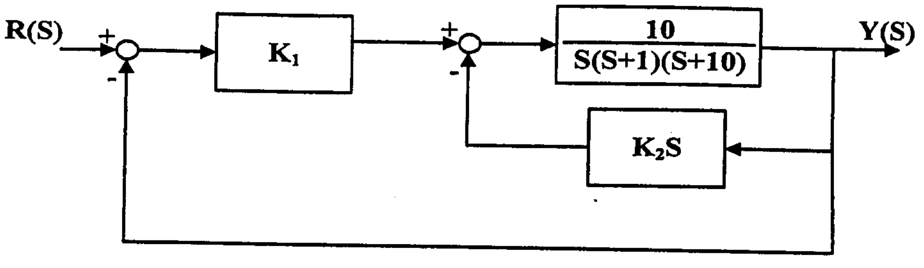

Fig. 3

4. (20%) Consider an open-loop plant $G(s) = \dfrac{1000}{s(s+10)}$ with a PD controller $C(s) = K_P + K_D s$ shown in **Fig. 4**. (a) (10%) Please design the controller so that the ramp-error constant $K_v = 1000$ and the damping ratio is 0.5. (b) (10%) Please implement the controller using the op amp circuit.

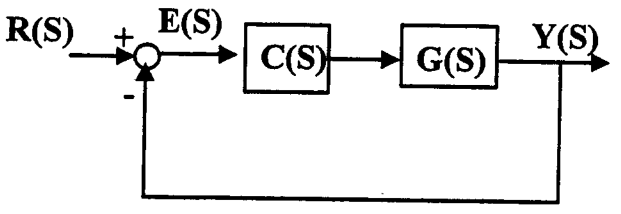

Fig. 4

5. (20%) (a) (10%) The state equations of a nonlinear system is

$$\dot{x} = y$$

$$\dot{y} = -|x|x - 2x - y^3 - 2 + 2\cos(2t)$$

Please find the equilibrium point (or operating point) of the system and find the linearized system at the equilibrium point.

(b) (10%) Consider the two nonminimum phase systems,

$$G_1(s) = \frac{-2(s-1)}{(s+1)(s+2)};\ G_2(s) = \frac{3(s-1)(s-2)}{(s+1)(s+2)(s+3)}$$

(i) (5%) Plot the unit step responses for $G_1(s)$ and $G_2(s)$, respectively.

(ii) (5%) Explain the differences in the transient behavior of the two responses as it relates to the zero locations.