

1. 15%

- (a) For a value in a decimal (十進位) system is 11.1, what is its expression in a binary (二進位) system? (5%)
- (b) Form part (a), how many bits does a computer require to store the binary digits for the decimal number 11.1? What will happen if a personal computer has only 32 bits to store the number? (10%)

2. 25%

We wish to solve $f(x)=0$ by using the Fixed-Point Iteration: $x_{n+1} = g(x_n)$.

- (a) Analyze the iteration error and derive the sufficient condition for the method to converge. (15%)
- (b) We wish to solve $f(x) = x^2 - 2x - 3 = 0$ by the $x_{n+1} = g(x_n)$ iteration. Is there a root in the interval $x \in [2, 4]$, why? (5%) If we write $x = \sqrt{2x+3}$ and $x = \frac{x^2 - 3}{2}$, which expression will converge? why? (5%)

3. 25%

Given the following set of data: $\frac{x}{f(x)} \left| \begin{array}{ccc} -0.1 & 0.0 & 0.2 \\ 5.3 & 2.0 & 3.19 \end{array} \right|$

- (a) Find the 2nd degree Lagrange interpolating polynomial passing through the above points. (10%)
- (b) Find the 2nd degree polynomial via the least square approach. (10%)
- (c) What is $f(0.1)$ from (a) and (b)? (5%)

4. 15%

Given the following set of data:

x	1	2	3	4	5
$f(x)$	2.4142	2.6734	2.8974	3.0976	3.2804

- (a) Calculate $f'(3)$ and $f''(3) = ?$ (10%)
- (b) Compute $\int_1^5 f(x) dx = ?$ (5%)

5. 20%

We want to integrate $\int_{-1}^1 f(x) dx$ numerically by the two-term Gaussian quadrature i.e.

$\int_{-1}^1 f(x) dx = c_1 f(t_1) + c_2 f(t_2)$. Find c_1, c_2, t_1 and t_2 . Use the above result to

evaluate $\int_0^{\frac{\pi}{2}} \cos x dx$ and compare the exact result.