本試題是否可以使用計算機：　□可使用　，　☑不可使用　（請命題老師勾選）

1. Suppose a fair coin is tossed n times. Show that the two events E1: "at least two tails" and E2: "one or two heads" are independent when $n=3$, but are dependent when $n=4$. (15%)

2. When transmitting a bit sequence, the bits could be flipped randomly due to the channel interference. Suppose the probability is p for a bit to be flipped from '0' to '1', and q for a bit to be flipped from '1' to '0' (i.e., $0 < p, q < 1$). If 30% of the bits to be transmitted are '0's and 70% of those are '1's, what is the probability that a received '1' is actually a '0'? (15%)

3. Given that $y_1=(1, 1, 0)$, $y_2=(2, 0, 1)$, $y_3=(2, 2, 1)$, transform y_1, y_2, y_3 to an orthonormal basis x_1, x_2, x_3 under the Euclidean inner product using the Gram-Schmidt orthogonalization process (beginning from y_1). (15%)

4. Use Gaussian elimination to solve the following system of linear equations. (15%)

$$\begin{cases} 9x_1 + 3x_2 + 4x_3 = 7 \\ 4x_1 + 3x_2 + 4x_3 = 8 \\ x_1 + x_2 + x_3 = 3 \end{cases}$$

5. Let A and B be two sets. Show that $A \cap B = (A-B) \cup (B-A)$. (10%)

6. Represent the postfix expression ((A (B C +) *) D /) as a binary tree. Also, give the corresponding prefix and infix expressions. (15%)

7. Let X and Y be two random variables with a correlation coefficient of $\rho(X, Y)=0.5$. Also, the standard deviation of X and Y are $\sigma_X = 3$ and $\sigma_Y = 4$, respectively. Find $\text{Var}(4X - 1.5Y + 3.89)$. (15%)