本試題是否可以使用計算機：☐可使用，☑不可使用　（請命題老師勾選）

1. Solve the following differential equation system by using matrix concept (20%)

$$\frac{d^2X}{dt^2} = -37X + 12Y$$

$$\frac{d^2Y}{dt^2} = 12X - 37Y$$

I.C.　$X(0) = 2$,　$Y(0) = 1$,　$\frac{dX(0)}{dt} = 1$,　$\frac{dY(0)}{dt} = 2$.

2. Verify the Stoke's theorem: $\iint \nabla \times \vec{F} \cdot \vec{n} dA = \oint \vec{F} \cdot d\vec{r}$ **(see Fig. 1) (20%)**

where the vector function : $\vec{F} = y\vec{i} - z\vec{j} + 3x\vec{k}$; surface S : $z = f(x,y) = 4 - (x^2 + y^2)$, $z \geq 0$

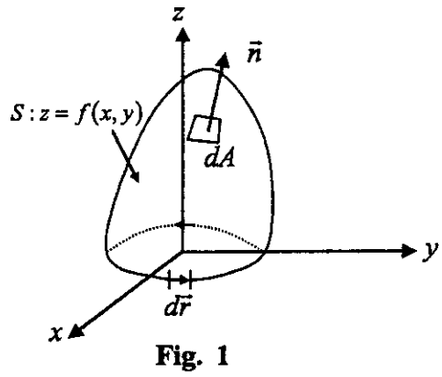

Fig. 1

3. Solve the following diffusion equation (20%):

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad \text{for } t > 0 \text{ and } 0 \leq x \leq L$$

I.C.　$T(x,0) = A\sin(\frac{n\pi}{L}x)$　　B.C.　$\begin{cases} T(0,t) = 0 \\ T(L,t) = 0 \end{cases}$

4. Solve the following wave equation (20%):

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{for } t > 0 \text{ and } 0 \leq x \leq L$$

I.C.　$\begin{cases} u(x,0) = 0 \\ \frac{\partial u}{\partial t}(x,0) = \delta(x - \frac{L}{2}) \end{cases}$　　B.C.　$\begin{cases} u(0,t) = 0 \\ u(L,t) = 0 \end{cases}$

where $\delta(x)$ is the delta function.

5. Evaluate the integral as following :

(a) $\int_{-\infty}^{\infty} \frac{\cos(mx)}{a^2 + x^2} dx$ for $m > 0$ (10%)　　(b) $\int_0^{\infty} \frac{x^{\frac{1}{3}}}{x(x^2 + 1)} dx$ (10%)